MARKETING SCIENCE
Vol. 10, No. 3, Summer 1991
*Printed in U.S.A.*

# MULTIPLE SOURCE PROCUREMENT COMPETITIONS

SUDHINDRA SESHADRI, KALYAN CHATTERJEE AND GARY L. LILIEN
*University of Maryland*
*Pennsylvania State University*
*Pennsylvania State University*

Relationships between buying and selling organizations in business markets are varied and complex. One important relationship is procurement, or the type of alliance that buyers form with sellers to fulfill their purchasing needs. Multiple sourcing is often proposed to prevent a variety of procurement problems. We develop a bidding model to investigate the effect of multiple sourcing on competitive behavior prior to supplier selection.

The model treats the number of bidders (and the decision to bid) endogenously. This distinctive feature allows us to show that the distribution of the number of bids is not independent of the competition type. Most previous models have not considered this question at all. Although multiple sourcing does increase participation in a bidding competition, we show that it can lead to strategic pre-award price increases that can mitigate the effects of the advantages stressed by most previous analyses of post-award supplier management.

**(Business Marketing; Equilibrium; Decision Analysis; Procurement)**

Market structures are fundamental to the analysis of marketing activity. The number and power of sellers, the number and power of buyers, the nature of the product, and transaction costs are among the important factors that dictate the structure of a market. For frequently purchased packaged goods, with a small number of powerful sellers, a large number of relatively homogeneous buyers, and with simple, low price (and low purchase risk) products, the familiar supermarket environment, with posted prices, works quite efficiently.

However, a large and growing portion of the annual procurement expenditures of the government (6.6 percent of the GNP or about $189 billion in fiscal 1988, according to a Census Bureau report) and much of the multi-trillion dollar business transacted between organizations is conducted via some form of procurement competition. In these competitions (of which the single-buyer, multiple-seller, one-time, competitive sealed-bid case is most frequently cited), potential sellers compete with one another, via the bid mechanism to "buy" business. In these markets, the buyer actually constructs the market by defining the rules of exchange (open vs. sealed bid, single vs. multiple sourcing, one-time vs. sequential procurement, etc.). Sellers, acting in their own best interests, respond via their bidding behavior or via their decision not to bid at all.

In this paper we investigate how source selection decisions affect the nature of competition in business procurements. Multiple sourcing, the splitting of an order among



0732-2399/91/1003/0246$01.25

multiple sellers, occurs frequently when buyers try to enhance competition by ensuring that several bidders share the contract. The buyer may also procure from different suppliers in turn, rather than simultaneously. The fundamental argument for multiple sourcing is that it should occur when the market could not otherwise support competition in the long run. But are buyers likely to find it effective?

There are three arguments commonly proposed in favor of multiple sourcing. The first has to do with whether the number of bidders who compete can be increased by reducing the risk of nonselection. Rothkopf (1983, p. 107) cites an example from the construction industry which makes explicit the importance of the number of bidders. In this example, a bid taker (buyer) was faced with an operational question of whether to delay a project in order to allow an additional bidder to prepare a bid or proceed with the best of the bids it had already received: "The cost of delay was easy to estimate, but what was the cost of doing without [an extra] bidder?" Multiple sourcing allows more than one supplier to succeed in the bidding competition and thus reduces this nonselection risk. What is the specific relationship between multiple sourcing and the number of bidders?

The second argument concerns the insurance a buyer needs against the possibility of stock-out. Webster (1984, p. 49) discusses the so-called stockless purchasing arrangement found especially in industrial components and subassemblies that tend to "lock in" organizations to their present vendors. Buyers face the risk that unanticipated surges in the procurement requirement could lead to crisis situations if there does not exist a pool of suppliers with installed capacity of collectively sufficient magnitude. What price does the buyer pay for this insurance?

Lastly we have two related but distinct arguments; first, that multiple sourcing offers the buyer the opportunity to manage supplier behavior after awarding the contract. The possibility of introducing some form of competition between the selected suppliers in order to provide incentives for post-award cost control exists only when multiple suppliers are chosen in the initial selection process (Seshadri 1990). Second, as Tirole states (1988, p. 27), collusion between personnel across firms in long-run buyer-seller relationships could lead to inefficient procurement and that might make a breach with existing suppliers desirable for a buyer. Cost escalation in government procurement is in part attributable to lack of alternate sources of supply: after the award is made and a monopoly situation evolves, prices tend to rise severely. Do strategic implications for competition for future contracts arise from supplier selection methods? This last argument for using multiple sourcing is best analyzed with a multi-period model, and this is beyond the scope of this paper (see Anton and Yao 1987; Raffel and Chatterjee 1989). However, we make some qualitative observations about the potential that multiple sourcing affords the buyer to benefit from post-award competition.

The model developed in this paper allows an explicit analysis of the first two arguments mentioned above. We derive the specific relation between multiple sourcing and the number of bidders. In addition, we show how to determine the overall expected acquisition cost to the buyer as a result of multiple sourcing. The buyer may trade off the increasing acquisition cost due to more suppliers against reduced risks of stock-out.

Finally, in addition to these multiple sourcing issues, we also investigate the seller's decision of whether or not to bid. The protracted and expensive process of bidding in response to a request for quotation (RFQ) often results in large opportunity costs for those sellers who the buyer rejects. The seller's decision to bid depends on how many others he thinks will also compete for the same business. What basis does the potential seller have for knowing how many rivals he is likely to bid against? Note the chicken-and-egg situation here: the number of rivals emerges from the process of all possible competitors wondering what each of their opposite numbers will decide. In this paper, we explicitly model this process and relate the process to the nature of the competition.

In the next section, we review some relevant models of bidding competitions. Then we present our model and discuss its key assumptions. Next, we outline the analysis of the model and derive our results, discussing them and relating them to some empirical findings and policy implications in the following section. We conclude by discussing the limitations and possible extensions of our analysis.

## Bidding Models

Hansen (1988) points out the fact that prices are set in all buying and selling situations by *someone* and a discernible *process* is used; one such process is competitive bidding. Many quantitative models have been developed as decision making guides in competitively bid procurement situations (Stark and Rothkopf 1979 provide an extensive bibliography; also see surveys by Engelbrecht-Wiggans 1980, and McAfee and McMillan 1987a). We will focus on the main features that distinguish our model from others, below (see Seshadri 1988, for a more complete review).

*Multiple suppliers.* Most bidding models assume that the buyer awards the winning contract to the single lowest bidder. An exception is Wilson (1977) who concludes that splits are counterproductive in share auctions. Although there are models of dual source procurement that demonstrate the role of competition subsequent to the award (Tirole 1988, p. 34; Anton and Yao 1987), hardly any analyze how split buys affect sellers' bidding strategies prior to the award. Our model differs from existing models of dual sourcing and share auctions in both intent and assumptions; we allow the buyer to select a predetermined number of suppliers from among a pool of potential sellers.

*Number of bidders.* The seller's decision to bid depends on what is known about the number of competitors. Individual sellers' decisions to bid are simultaneous if sellers are symmetric and must, therefore, be made without perfect knowledge of the final number of committed bidders.

Several researchers have examined the number of bidders issue. McAfee and McMillan (1987b) model a bidding competition where the number of bidders is known stochastically, but the process that determines the probability of any given number of bids is exogenous. In another model (McAfee and McMillan 1987c), they determine the number of bidders by setting an economic profit equal to zero, as in a competitive market with free entry.[1]

Engelbrecht-Wiggans (1987) develops a relationship between the buyer's declared reservation price and the number of bidders. In his model, the probability that a given number of bidders bid has a distribution related to the competition type, but the specific relationship to a seller's expected profit is not made explicit.

Samuelson's (1985) model has each seller observing his cost prior to deciding to bid. A firm bids only if its cost is below some specified level. However, he does not consider the effect of split buys.

Finally, Hansen (1988) speculates that when the number of bidders is endogenous, any competition that seeks to attract sellers by offering greater profits could well result in smaller seller profits due to the increased competition engendered, leading, perhaps, to a possible "revenue equivalence theorem" much more general than any existing theory. The endogenous determination of the number of competitors is a key characteristic of our model.

*Seller characteristics.* Most bidding models assume that sellers base their bids on some privately known characteristic, such as their cost structure, their need for the

[1] A referee has drawn our attention to Harstad's (1990) independent work on a common value auction model with a mixed strategy participation equilibrium. As Harstad himself writes in footnote 6 of his paper, his results may not apply to a private value model like ours.

business, etc. The models then formulate a bidding strategy as a function that relates a seller's characteristic to his bid. An important distinction exists in the literature depending upon whether private characteristics, usually modeled as random variables with a commonly known distribution, are independent or correlated. If they are independent (Maskin and Riley 1984, for example), the model is an independent private values model. If correlated in a particular way (Milgrom and Weber 1982, for example), it is an affiliated private values model. In the latter model type, a situation called the winner's curse (Capen, Clapp, and Campbell 1971) may occur whereby the seller that wins the contract may win precisely because he has underestimated his costs and, hence, bid too low to earn a profit. Here, however, we concentrate on the independent private values approach.

*Bid basis.* The characteristic on which the bid is based has been modeled either as the estimated cost for the contract or as the estimated cost plus opportunity cost. McAfee and McMillan (1986, 1987a) and others use the first approach, where the seller's bid is a perfect signal of his cost estimate for the contract. Samuelson (1986), McCall (1970), and Canes (1975) use the second approach. Our model combines these approaches by including opportunity cost in the decision to bid, while modeling the bid itself as a function only of estimated cost.

We demonstrate below that the endogenous determination of the seller's decision to bid provides key insights into the role of the competition type (such as multiple sourcing) in competitive procurement. We now formalize the key assumptions of our model.

## A Model of Multiple Source Procurement

### *Outline and Example of the Procurement Process Being Modeled*

The XYZ company wants to standardize its word processing system company-wide and is looking for suppliers. Assume there are 25 potential manufacturers. A buyer for XYZ puts out a Request for Quotation (RFQ) and ten sellers respond by ordering copies of it. The number of *potential* bidders is therefore ten. The remaining 15 sellers (whom we call *nonstrategic sellers*) determine the price at which the buyer could award a noncompetitive contract. This means that the buyer could pick a supplier from among those qualified sellers not interested in competing by offering to negotiate a price with one of them. The buyer uses a competitive procurement mechanism, such as a sealed bid procedure, because he expects to satisfy requirements at a price lower than that which he could obtain from a nonstrategic seller. We call the noncompetitive price the buyer's *reservation price*.

Assume also that the RFQ specifies that two suppliers will be awarded equal portions of the contract, at the same price. This means that the buyer will not discriminate between the suppliers finally chosen. (In general, the buyer may select any number of suppliers and employ discriminatory competition types.) Of the ten potential bidders, say four decide to bid. The remaining six who examined the contract but did not bid chose not to commit capacity or devote the necessary time and effort to preparing a competitive bid. The four who did decide to bid now develop a design, begin to gather the necessary cost data from subcontractors, etc., and select a bidding strategy. The process of bid preparation often calls for activities, such as search for subcontractors that reveal the bidder's identity to other similarly committed bidders. Each of the bidders is now better able to estimate their probability of selection and their profit at each bid price.

Once the buyer receives and evaluates the bids, the bidders learn the identity of selected and rejected suppliers. The two (out of four) bidders whom the buyer rejects make no profits from their decision to participate. However, the two selected suppliers fulfill the contract and then realize their profits. Those profits depend on the suppliers' actual costs (not on their estimated costs) as well as on the contract price.

What if only one seller decides to bid? Since the RFQ states that the buyer will select two suppliers, in this circumstance the buyer's attempt to introduce competition fails and noncompetitive procurement results. The buyer has several options, including awarding the whole contract to the single bidder and adding one or more of the nonbidders as a supplier. In our model, we assume the buyer selects the single bidder and awards him the promised one-half portion of the contract at the reservation price. He awards the rest of the contract to one or more of the nonstrategic sellers (one of the 15 who did not respond to the RFQ). If no bids are received, we assume the buyer awards the entire contract at its reservation price to one or more of the nonstrategic sellers.

Figure 1 depicts the process outlined above. The sellers' decisions are first whether to bid or abstain ($S1$) and next what price to bid ($S2$). The buyer makes the evaluation decision ($B1$) after obtaining the bids.

In this process there are several key uncertainties and information asymmetries. We now define our model assumptions.

*Assumptions*

For the procurement process outlined above, we first motivate and then formulate each key assumption.

*Number of bidders.* The buyer keeps an open list of sellers who can request a copy of the RFQ. These are called qualified or potential suppliers. We assume that the buyer and all the sellers know the maximum number of bids that the buyer may receive in a particular industry. Thus,

A1. The number of potential bidders is commonly known.

*Selection criterion.* Price is frequently a key criterion for selecting a supplier. In some procurements where quality is closely specified or standard and other terms such as delivery dates exhibit only minor variations, the only important factor in the competition is price. Often the buyer publishes the formula he uses to transform other terms of the contract to monetary equivalents for the purpose of evaluating bids. Therefore, we restrict our attention to pricing strategies.

A2. Sellers bid a price for the contract and the buyer awards the contract based on the lowest bid price or prices.

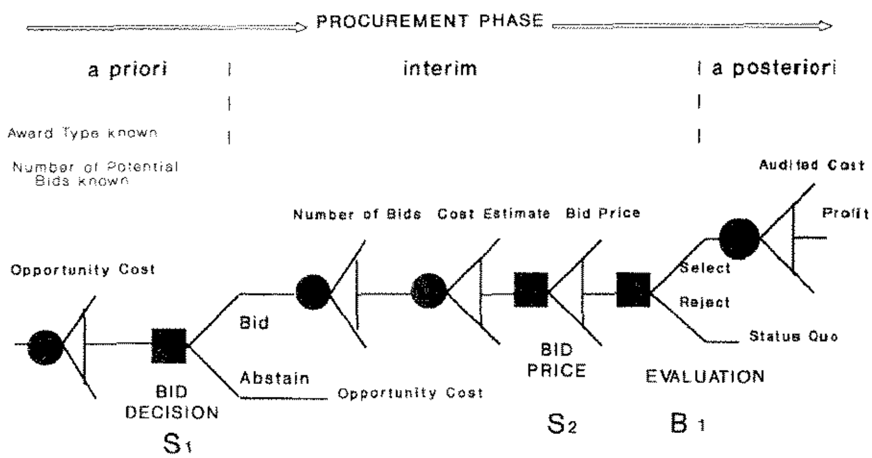


FIGURE 1. The Procurement Decision Process.
The procurement process consists of a sequence of decisions (block nodes) and uncertain events (circular nodes). The sellers' decisions are $S_1$ and $S_2$; the buyer's decision is $B_1$.

*Sellers' bid decision rules.* Sellers in procurement markets usually represent medium to large size firms where risk attitudes are not easy to determine. A reasonable view, then, is that sellers should be expected profit maximizers. Then sellers could view each procurement separately and incrementally above some ongoing level of business.

Sellers may adopt dissimilar strategies when they bid for procurement business. Differences arise when certain sellers behave idiosyncratically, perhaps by bidding very low to win a contract and thus become qualified bidders for future contracts. In many procurements, bidders seek to exploit specific differences between their bids and the bids of potential rivals. Such differences of strategic importance arise primarily due to technological differences between sellers. Only free access to technology would serve to limit the strategic importance of technological differences among sellers.

It is therefore restrictive in a number of procurement situations to assume that sellers are *strategically* symmetric. However, this assumption is made in most bidding models. A rationale is provided in a theorem by Maskin and Riley (1984) that symmetric strategies are the *unique* equilibrium for the bidding model that we analyze in this paper, where private information is distributed on a bounded support. (Intuitively, symmetric strategies assume that identical players play identical strategies.)

Assumptions 3 and 4 (although the preceding discussion established that the latter is a strong assumption) summarize the sellers' strategic decision rules for our model.

A3. The sellers' profits from the procurement activity are incremental over their routine profits from alternative business. Firms seek to maximize incremental expected profits.

A4. The only relevant differences between sellers are their (privately known) opportunity costs and cost estimates.

*Seller characteristics.* Procurement markets are typically those for which the seller needs *dedicated* production facilities or skills to prepare technically competitive bids. The seller often commits the design and production capacity that he would use for the specific procurement (if he were selected as a supplier) at the time the decision to bid is made. The seller may make irretrievable investments in technology in order to determine state-of-the-art production costs or incur option-type expenses with subcontractors. Often the seller must guarantee the bid as valid over several months during the evaluation phase. Rogerson (1988) discusses the nature of some of these expenses that qualify sellers to bid for contracts. The cost of maintaining commitment-free capacity can be significant. Each seller has to assess his opportunity cost prior to his decision to bid. The opportunity cost of committing capacity depends upon the particular seller's aggressiveness in landing business as well as the industry's volume of business activity: larger opportunity costs are more probable as industry capacity utilization rises. While information on competitors' opportunity costs or their likely cost estimates are closely guarded and secret, sellers have both the market intelligence and expertise to develop estimates of what those competitive costs are likely to be. Thus, we assume that all sellers have equal access to this market intelligence and that any individual seller's private information is a random variable that is drawn from a known distribution, with only that particular seller knowing its true value. This approach follows from Harsanyi (1967, 1968a, b) and many later researchers. While Harsanyi's treatment allows for the distribution to be conditioned upon the privately observed characteristic, for analytic tractability we assume the distributions are independent of the observed information of the individual seller (see McAfee and McMillan 1987a, for example).

It also simplifies our analysis if the distributions from which the sellers' private information are drawn are identical. (This is not to say that the private information itself is the same across sellers.) This is a restrictive assumption since differences in economic terms and conditions, such as financing or delivery schedules, often arise. Buyers usually translate these economic differences into their monetary equivalents. For example, government buyers sometimes favor local suppliers over foreign suppliers with a 6% price differential advantage. This assumption of stochastically identical sellers is clearly a simplifying assumption that is not appropriate in many situations. However, the existence of stochastically identical sellers is frequently assumed (see McAfee and McMillan 1987a). A partial justification is that persistent differences between sellers will cause disadvantaged sellers to depart from active competition before too long. Similarly, we will treat costs prior to estimation and auditing as random variables with common, known, and independent probability distributions.

The arguments above are summed up as follows:

A5. A seller's actual characteristics—audited cost, estimated cost, and opportunity cost—are independent of any other seller's characteristics. Sellers draw such costs independently from identical probability distributions for characteristics that are uncertain (i.e., sellers are stochastically independent and identical).

These assumptions and the procurement process outlined above allow us to pursue our analysis.

## Model Analysis

We are now in a position to seek rigorous answers to the questions we raised at the outset. As we saw before a central question is how multiple sourcing influences the sellers' likely bidding behavior. Thus, we begin by deriving the seller's bidding strategy. We then determine how it relates to the seller's expected profit at various points in the procurement process. Next, we show how the decision to bid and the number of bidders is endogenously determined in our model. Finally, we relate the buyer's price to sellers' bids and profits under multiple source procurement.

### *Analysis*

In this section, we relate the buyer's sourcing decision to the seller's bidding strategy, expected profits, and decision to bid, and derive price consequences for the buyer.

We list our key notation in the order in which the terms are introduced into the analysis. We adopt the following convention: letters of the alphabet from $A$–$K$ represent constants; from $L$–$Z$ they represent random variables. Lower case letters generally signify values of the random variable.

$B$ = number of potential bidders,
$N$ = number of actual bidders,
$S$ = bidder's cost estimate, before producing the goods,
$J$ = bidder's bid price,
$F(s)$ = probability distribution function of $S$,
$K$ = number of suppliers to be selected,
$L$ = the $K-1$th lowest of $N-1$ bid prices,
$M$ = the $K$th lowest of $N-1$ bid prices,
$V$ = the $K-1$th lowest of $N-1$ cost estimates,
$W$ = the $K$th lowest of $N-1$ cost estimates,
$Z(s)$ = bidding strategy function that assigns a bid price to a value of the cost estimate,
$H(v, w)$ = joint probability distribution function of $V$, $W$,
$R$ = the seller's opportunity cost, and
$G(r)$ = probability distribution function of $R$.

*Bidding strategy.* Now, of $B$ potential bidders, only $N$ actually bid. We focus on the $i$th of the $N$ bidders, whose cost estimate is denoted by $S$, and whose bid price is $J$. Let $F(\cdot)$ be the probability distribution function of $S$. We place the bids of the remaining $N-1$ bidders in ascending order. Of these $N-1$ bidders, the $K-1$th lowest bid is $L$, and the $K$th lowest is $M$. These remaining $N-1$ bidders are assumed to use a common bidding strategy, which is a function of their cost estimates and whether or not the number of bidders is more than the required number of suppliers. The bidding strategy function, $Z(\cdot)$, is assumed to be continuous and strictly increasing and therefore invertible. We then define the *inverse bidding strategy* as $Z^{-1}$ (bid price) = cost estimate, i.e., $Z^{-1}(l) = v$, $Z^{-1}(m) = w$, and, in general, denote $Z^{-1}(J) = x$. It is our objective in this part of the analysis to derive the equilibrium bidding strategy, $Z(\cdot)$.

At this stage we need to ask how the contract price is determined. In this paper, we consider the nondiscriminatory multiple source competition which sets the uniform contract price at the highest accepted bid. Other possible pricing schemes such as the lowest rejected bid (Vickrey 1961, 1962; Rothkopf et al. 1990) or discriminatory bids (Ortega-Reichert 1968; Weber 1983) also fit our model. Analysis of these pricing schemes in multiple unit auctions, such as those for Treasury Bills (Ortega-Reichert 1968; Weber 1983), demonstrate that the expected revenues to the seller remains the same regardless of the pricing scheme. This result is familiarly known as revenue equivalence. The multiple unit auction models assume that the number of bidders is *not less* than the number of units auctioned. Our model relaxes this assumption. However, we find that revenue equivalence still holds, in that the buyer's expected cost remains the same across these pricing schemes, provided the number of suppliers to be selected remains constant.

Our next step is to relate the seller's expected profit from the interim phase of the competition to the adopted bidding strategy. Let us first examine the sellers' bidding strategy when the number of bidders is more than the required number of sellers. This is the event where $K < n \leq B$. We now proceed to derive the equilibrium bidding strategy in a manner similar to Wilson's (1982) analysis of double auctions. The $i$th bidder's bid will determine whether he is selected as a supplier and what the contract price will be if he is selected. There are three possibilities:

(i) If $Z^{-1}(J) = x$ satisfies $v < x < w$, then the price is $J$, since the $i$th bidder will be the $K$th lowest of all $n$ bidders.

(ii) If $x \leq v$, the $i$th bidder will not necessarily be the $K$th lowest bidder, but will be selected among the $K$ lowest bidders. The contract price will be $l$.

(iii) If $w < x$, the $i$th bidder will not be selected.

The $i$th seller's expected profit from the interim phase of the competition depends upon his cost estimate ($s$) and the number of actual bidders ($n$), both of which are known by now. If we call $H(v, w)$ the joint probability distribution function of $V$ and $W$, then the above three possibilities will yield an expected profit (with the expectation taken over the probability of selection) which can be written as

$$\pi_K(s, n) = \int_{l<J\leq m} (J - s)\,dH(v, w) + \int_{J\leq l} (l - s)\,dH(v, w). \tag{1}$$

It is straightforward to change the variable of integration in (1) with the inverse strategy function, $Z^{-1}(\cdot)$, to yield the relationship between expected profit and bidding strategy:

$$\pi_K(s, n) = \int_{v<x\leq w} (Z(x) - s)\,dH(v, w) + \int_{x\leq v} (Z(v) - s)\,dH(v, w). \tag{2}$$

The $i$th seller's optimal bid price, $J^*$, maximizes his expected profit $\pi_K(s, n)$. Since $\pi_K(s, n)$ is concave in $Z(x)$ (or $J$), we may use the first-order condition for optimality on (2) to determine $J^*$, $d\pi_K(s, n)/dZ(x) = 0$. Thus,

$$H(x) + (Z(x) - s)\frac{dH(x)}{dx}\frac{1}{Z'(x)} - (Z(x) - s)h(x)\frac{1}{Z'(x)} = 0, \qquad \text{where}$$

$H(x)$ = probability that $x$ lies between $v$ and $w = \int_{v<x\leq w} dH(v, w)$,
$h(x)$ = probability density function of the $K - 1$th lowest of $n - 1$ bids.

How may we simplify this equation to determine the bidding strategy, $Z(\cdot)$? Our behavioral assumption of symmetric strategies (A4) comes into play here. The same strategy $Z(\cdot)$ is optimal for *all* bidders when no bidder has an incentive to *unilaterally* deviate from using $Z(\cdot)$. This is the familiar, symmetric *Bayesian Nash equilibrium* concept. The Nash equilibrium requires that the $i$th bidder also find it optimal to use $Z(s)$ when all other bidders are known to be using $Z(\cdot)$; thus, the first-order condition for optimality must also satisfy $x = s$, and $J = Z(x) = Z(s)$, for the equilibrium. Hence, we obtain the differential equation that a bidding strategy, $Z(\cdot)$, must solve in order to be in equilibrium:

$$Z'(s)H(s) + (Z(s) - s)\left[\frac{dH(s)}{ds} - h(s)\right] = 0. \tag{3}$$

A solution to (3) is a bidding strategy that relates a bid to every cost estimate. This bidding strategy may be used in (2) to give us the bidder's expected profit, $\pi_K(s, n)$. We next carry the analysis through to determine the expected profit over the additional uncertainties of cost estimates and the number of bidders.

*Expected profit.* The above analysis derived conditions for the equilibrium bidding strategy and the bidder's expected profit when the number of bidders is known to be *more* than the required number of suppliers. If this is not the case, i.e., if $1 \leq n \leq K$, then all bidders are selected and each bidder makes the largest possible profit, $\pi_K(s, k)$. Here, competitive procurement has failed and the buyer is forced to buy at his reservation or noncompetitive price. We may therefore write the bidder's expected profit from the competition as

$$\pi_K(s, n) = \begin{cases} \pi_K(s, n), & K \leq n - 1 < B, \\ \pi_K(s, k), & 0 \leq n - 1 < K. \end{cases} \tag{4}$$

We now address the uncertainties bidders face during the interim phase (Figure 1) of the procurement. Sellers differ on their expected profits once they know their estimated costs; however, the expected profit before they learn their estimated cost is the same for all sellers, since they all face the same distribution of estimated costs by assumption A5. With the expectation of profit taken over the distribution of cost estimates, the expected profit prior to learning their cost estimate, denoted by $\pi_K(n)$ for every seller, is

$$\pi_K(n) = \int_s \pi_K(s, n)dF(s). \tag{5}$$

We denote a seller's expected profit just before learning the number of bidders by $\pi_K$. This a priori expected profit depends on the probabilities with which any given number of sellers bid. Thus,

$$\pi_K = E[\pi_K(n)] = \sum_{n=1}^{B} \pi_K(n)\Pr\{N = n\}. \tag{6}$$

How are the bidding probabilities, $\Pr\{N = n\}$, determined? In our model, all sellers decide whether or not to bid simultaneously; this is the key to determining bidding probabilities. We now elaborate on this key step and establish the (endogenous) relationship between bidding probabilities and expected profits.

*Number of bids.* In order to derive the bidding probabilities, we must first ask when a seller should bid. The decision rule we propose is that the seller should bid if and only if its a priori expected profit is no less than its opportunity cost, $r$; i.e., if

$$\pi_K \geq r. \tag{7}$$

This decision rule or strategy has an important property. When other sellers are known to be using this strategy, a seller has no incentive to unilaterally deviate from using the same strategy, which is therefore a Nash equilibrium strategy. Since we have used backward induction in deriving this equilibrium, it is also clearly subgame perfect. Thus, a seller should use this decision rule as his best strategy, expecting to use the bidding strategy $Z(s)$ later when called upon to bid and should actually do so once that stage of the game is reached.

With this answer as to when the seller should bid, we can derive the bidding probabilities. If $\pi_K$ is greater than any feasible value of $R$, all sellers will bid and $N = B$. The more interesting case is when $\pi_K$ lies within the range of feasible values of $R$. Denote the probability distribution of a seller's opportunity cost for the competition type where $K$ suppliers are to be selected as $G(r)$. The probability that a seller bids is therefore, from (7), $\Pr(r \leq \pi_K) = G(\pi_K)$. This probability is the same for every seller since $\pi_K$ is the same across sellers, and so is $G(r)$ by assumption A5. Each seller can now view each competitor's decision to bid or not to bid as a Bernoulli trial, with the probability of a bid given by $G(\pi_K)$. The number of bids is therefore a binomial random variable with a probability mass function given by

$$\Pr\{N = n\} = \binom{B}{n}[G(\pi_K)]^n[1 - G(\pi_K)]^{B-n}, \qquad 0 < n \leq B. \tag{8}$$

We can rewrite (6) for the a priori expected profit as

$$\pi_K = \sum_{n=1}^{B} \binom{B}{n} \pi_K(n)[G(\pi_K)]^n[1 - G(\pi_K)]^{B-n}. \tag{9}$$

Since $\pi_K$ appears on both sides of (9), the conditions for the existence of a solution to (9) must be established. As demonstrated in the Appendix, a solution to (9) will exist if $\pi_K(n)$ on the right-hand side of (9) is decreasing in $n$. This follows since for known competitions the expected profit $\pi_K(n)$ will fall when the number of bidders, $n$, increases. We therefore see that the probability of any given number of bidders and the seller's expected profits are uniquely related via (8) and (9).

Now we can investigate the consequences of these endogenous bidding probabilities for the buyer.

*Buyer's price.* The buyer's price (sometimes called acquisition cost) is the total payment to suppliers. Denote the buyer's expected price during the interim phase (but after the number of bidders is known) by $T$. Note that the case where the number of bidders does not exceed the required number of suppliers (including $n = 0$) results in the maximum or reservation price for the buyer, and call this reservation price $T_{\max}$. Thus, $T$ is given by

$$T = \begin{cases} KE[Z(s_k)], & K < n \leq B, \\ T_{max}, & 0 \leq n \leq K, \end{cases} \quad \text{where} \tag{10}$$

$Z(s_k)$ = bid of $K$th lowest of $n$ bidders,
$T_{max}$ = buyer's reservation price.

The buyer's a priori expected price is $E[T]$, where the expectation is taken over the number of bidders. This expectation may be written in terms of whether the number of bidders exceeds the number of required suppliers as follows:

$$\begin{aligned} E_N[T] &= E_N[E[T|n]] \\ &= E_N[E[T|n:K < n]] + E_N[E[T|n \leq K]]. \end{aligned}$$

Thus,

$$E_N[T] = E_N[E[T|n:K < n] + T_{max} \Pr\{n \leq K\}. \tag{11}$$

The buyer's a priori expected price, $E[T]$, consists of two terms, one conditional on the event that the number of bids is greater than the required number of suppliers, and the other on its complement. We know the probabilities of these events from (8) and (9), so we may determine $E[T]$ above. The buyer now knows how the number of suppliers he chooses, $K$, influences his price via the number of bidders and their bids.

The connection between multiple sourcing and its consequences (such as the number of bids, the seller's profits and the buyer's price) are now clear, in a probabilistic sense. We now turn to the implications of these relationships.

## Discussion

In this section, we compare our theoretical results with some related empirical findings and discuss the strategic issues raised by our model for buyers and sellers.

### *Empirical Comparison*

In their analysis of data on risk and return from the aerospace industry, Greer and Liao (1986) developed a multiplicative model relating average contract price to quantity bought, capacity utilization, whether the buy was a noncompetitive procurement, whether or not it was a winner-take-all competition and whether or not it was a dual source competition. The results of our model and the findings of Greer and Liao have much in common.

Greer and Liao report that as the capacity utilization rate rises, the contract price rises. Does our model provide a consistent prediction? We must first recognize the parallel between the empirically determined capacity utilization rates and the opportunity cost distribution of our model: high capacity utilization means that the industry is busy and high opportunity costs are probable, and vice versa. Shifts in the opportunity cost distribution, $G(r)$, mirror changes in the industry capacity utilization rate. The model does indeed show that a shift of $G(r)$ to the right usually increases the probability of fewer bidders and leads to higher expected acquisition costs. (This shift in $G(r)$ to the right corresponds to change of $p$ to smaller values in Figure A1.)

Greer and Liao find that winner-take-all awards result in lower contract prices than do dual source awards, all other variables held constant. The theoretical equivalent to this empirical situation is the case where as a direct result of multiple sourcing the opportunity costs distribution, $G(r)$, did not shift to sufficiently low values when compared to the shift in the contract cost distribution, $F(s)$. When $G(r)$ shifted to the left the probability of greater numbers of bidders increased, but not to the extent required for lower prices. In this case, more competition did not compensate for selecting a less efficient supplier along with the lowest bidder. This may well be the usual case, since economies of scale will not usually allow $F(s)$ to shift proportionately to the split in the contract size.

Will multiple sourcing ever result in a lower acquisition cost for the buyer? For this to happen, the probability of more bidders must increase to the degree that its effect on bid prices more than compensates for the inefficiency that the selection of higher cost suppliers will introduce. This depends on the specific distributions for $F(s)$ and $G(r)$ and the economies of scale. The result from (8) and (9) provides both the buyer and sellers with a basis for evaluating the level of competition different competition types are likely to engender. Multiple sourcing affects the distributions of contract costs, $F(s)$, and opportunity costs, $G(r)$. Splitting the contract into several ($K$) portions reduces the range of the distributions and shifts both $F(\cdot)$ and $G(\cdot)$ to the left, but to different degrees. In fact, the effect of multiple sourcing depends critically on the relative magnitude of the shifts. The following argument clarifies this finding.

If reducing the size of the contract by a factor $1/K$ shifts $G(\cdot)$ to sufficiently low values then $G(\pi_K)$ will be higher for given $\pi_K$. As demonstrated in the Appendix, this increases the probability of higher numbers of bidders. Of course, we must keep in mind that $\pi_K$ will also be smaller when the contract is split. This may be verified by noting that $\pi_K$ depends on $F(\cdot)$ in (5) and that $F(\cdot)$ also shifts to lower values. Therefore, just how many more bidders there are depends upon the relative magnitude of the shifts in $F(\cdot)$ and $G(\cdot)$. The overall effect on the buyer's price can be calculated from (11), though we cannot tell merely by inspection which direction the price will move. However, the qualitative discussion above makes the effects clear.[2]

Greer and Liao report that a capacity utilization rate above 80 percent results in a noncompetitive procurement situation. Our model handles the phenomenon in equation (11), the expected buyer's price equation for $E[T]$. When the mean opportunity cost becomes sufficiently high, the event $\{n \leq K\}$ dominates and competitive bidding ceases to be effective as a price reducer.

### *Strategic Issues*

Our modeling approach offers some insight into buying and selling behavior.

The *industry's propensity to compete* is determined by opportunity costs. Greer and Liao used this term to reflect the critical effect of industry capacity utilization rates on the buyer's price. Our analysis yields the opportunity cost, $R$, which is correlated with industry capacity utilization, as an alternate measure of the propensity to compete. We have confirmed that the propensity to compete is certainly crucial, but it is not the only element that drives competition.

A second crucial element is the *industry's potential to compete*, determined by the number, $B$, of potential bidders who comprise the pool of interested sellers able to make strategic bids. The effect of this potential to compete has generally been ignored until now, as researchers have focused on the actual number of bidders, $n$. Not all potential bidders will actually bid, but, as $B$ gets larger, the procurement is likely to be more competitive. Our analysis quantifies this likelihood and highlights the importance of the industry's potential to bid. The quantities $R$ and $B$ need not be related, for example, if a firm's foregone opportunity is in a market not serviced by the other $B - 1$ potential bidders.

The discussion above suggests that participants in procurement markets may benefit from focusing on the following issues:

[2] The details of the calculations for these tradeoffs for a uniform distribution are available as an appendix from the authors, and are not included here for reasons of length.

(1) The conventional wisdom is that buyers should seek better performance when procuring from multiple suppliers by raising the possibility of switching between them, by offering them different shares of the business, or by otherwise making them compete. The reviewed literature on dual sourcing demonstrates this possibility in the post-award period once the suppliers are selected. Our model, however, focuses on the pre-award period of the procurement process. Our analysis shows when sellers will strategically increase their bid prices under multiple sourcing bidding competitions. This happens whenever the increase in competition due to more likely bidders does not compensate for the decrease in selection risk due to multiple sourcing. We also show the net effect on the buyer's price (acquisition cost) and the conditions under which it increases. This happens whenever the net change in bid prices due to changed competition and selection risk does not compensate for the decrease in efficiency due to selection of higher cost suppliers. When viewed as two *independent* steps in the procurement process, therefore, the economic advantage of post-award competition that becomes possible when multiple sourcing is used will partially be lost due to the sellers' strategic increases in bids, and increases in the buyer's expected price during the pre-award competition.

The literature on dual sourcing has hitherto viewed the two periods as independent. In some situations the two periods may legitimately be viewed as independent and the modeling may be decoupled. This is possible when the buyer is pre-committed to selected suppliers, and selection is a nonissue. It may also be argued that when it is difficult for sellers to predict the rules that will be in effect during the post-award competition, it is appropriate to model their behavior as myopic. Therefore, they will have to adopt a nonstrategic perspective on the post-award period when they actually decide whether to bid, and bid, during the pre-award period. Our result of the trade-off between the two periods with respect to the buyer's cost holds *even when the two periods are viewed as independent*.

Normally, though, the two periods are dependent. The procurement process involves both selection of suppliers and their control. The buyer may also pre-commit to a particular form of post-award competition and sellers could take strategic advantage of their knowledge of this pre-commitment when bidding. The strategic dependence of the two periods is likely to further mitigate the buyer's post-award advantages from multiple sourcing. This could be analytically established with a multiperiod model that links the strategies available under the two different competitive situations.[3] A multi-period model that incorporates learning has recently been investigated by Raffel and Chatterjee (1989).

(2) For the kinds of procurements considered in our model, dedicated capacity is critical. A few suppliers relative to the volume of the procurement business indicates limited slack capacity. The objective in increasing dedicated capacity is usually to insure the buyer against sudden surges in demand. Our analysis shows that this insurance is possible through multiple sourcing. Thus, the greater bidding probabilities under multiple sourcing imply that more sellers should decide to give up alternative business opportunities and invest in the capacity required for the procurement business. But this insurance could come, as our analysis shows, with an increase in the buyer's price per individual procurement. If the long-term procurement cost includes stockout penalties that buyers pay due to insufficient capacity among suppliers in addition to the buying price, then there is a rationale for split buys on grounds of long-term procurement cost minimization. Our analysis quantifies the specific tradeoff the buyer faces between short-term price control and longer term competition resulting in supply assurance.

Some further implications provide a nice confirmation of the conventional wisdom.

(3) There are several insights that follow from the manner in which the number of bidders is determined in our model. First, if the number of bidders was considered fixed

[3] We are indebted to an anonymous reviewer for stressing this point.

and known we could not have recognized the advantage noted above of multiple source procurement over winner-take-all awards in increasing slack capacity, by increasing the likelihood of more bidders. By relaxing this assumption we can highlight this advantage. Second, if the number of bids was considered exogenously distributed the buyer would have to be a passive observer of the number of bids, which he could not influence. By relaxing this assumption we could show how the number of chosen suppliers influences the number of bids, via the seller's profits. The buyer can actively manipulate the number of bidders with multiple source procurement to suit his objectives. Another way the buyer may manipulate the number of bids for a fixed number of suppliers is by either charging the vendor a fee in order to submit a bid or by subsidizing the vendor's bid preparation costs. The buyer can factor in the change these fees and costs will mean to the seller's expected profit when estimating the influence on the number of bidders. Thirdly, if we used a zero-economic profit criterion to determine the expected number of bids we would not have obtained a distribution of bidding probabilities. This is the approach taken by papers reviewed earlier where bidders enter the competition until further entry will lead to negative expected profits. By relaxing this assumption we are able to assign definite probabilities to various numbers of bids. This last difference is related to another important issue. Rogerson (1988) poses the question that if expected economic profits are going to be zero then what is the firm's incentive in the first place to participate in the procurement business? He answers the question by demonstrating the existence of a "prize" won by the winning firm in its stock market value. Our approach to determining the distribution of the number of bidders does not assume zero economic profit, and therefore provides the more direct answer: firms can expect to make real economic profits from winning the procurement competition. Finally, we learn that sellers should incorporate the uncertainty about the number of bidders in their decision whether to bid or not. They have a way around the chicken-and-egg problem of which comes first: the decision to bid or the number of bidders. The seller may determine his a priori expected profit simultaneously with the bidding probability and arrive at a better decision.

(4) The seller should seek market intelligence on opportunity costs and estimated costs of competitors. The number of firms on the buyer's bid list is important information. Sellers should attempt to keep this number to a minimum. Webster (1984, p. 177) notes that "a common selling strategy is to get procurement specifications developed in such a way that the seller's product offering will be favored." Our analysis also suggests that specifications that narrow the number of potential sellers will serve the seller's interests.

(5) The buyer can reduce the price paid by procuring in periods when the industry has a higher propensity to compete. The buyer may also reduce the price by qualifying a greater number of sellers in order to increase the potential to compete. This may be accomplished by writing broad specifications that allow more potential bidders to qualify. Webster (1984, p. 177) notes that "procurement managers will strive hard to avoid specifications that de facto result in a sole source or limited competition."

## Conclusion

We now sum up our key contributions, stress the main limitations of the model and suggest some extensions.

### *Contributions*

In summary, most existing models have either ignored split-buy procedures or asserted that they are suboptimal. This is because their analysis proceeds with an exogenously given number of bidders. We derive the number of bidders endogenously and make clear the buyer's trade-off, in a single period, between insuring with multiple suppliers against a variety of single sourcing problems and obtaining the lowest expected price.

*Limitations*

Procurement is a complex activity, and our model has many limitations, of varying importance to our main results:

(1) Correlated cost estimates, which we did not consider here, are an important issue for any bidding competition. However, the risk of the winner's curse introduced by correlated cost estimates is not reflected in our results which are driven by the risk of losing the contract.

(2) Split buys are complicated by the presence of economies of scale, but our model has not dealt with this. The introduction of economies of scale would increase the realism of the model.

(3) An important issue is the use of unequal splits in multiple source awards. Often, the portion of the contract awarded to a supplier depends upon his bid. This will certainly introduce additional strategic considerations into the seller's bidding decisions. Boger and Liao's (1988) study of step-ladder bids, where sellers bid prices for various portions of the whole contract, indicates that sellers inflate bids for portions other than those they seek. This could indicate that equal splits may have some advantages over splits that allow bids to vary with portions of the contract, but further research is needed to study this point.

(4) Finally, real procurements have a wide range of situation-specific characteristics (heterogeneity of bidders in terms of costs, capabilities, objectives, strategies, other business and the like) that are difficult to include in any analytic model. Our model is no exception, and the assumptions of stochastically identical sellers and strategic symmetry should be relaxed for greater realism. Based on our reading of earlier auctions literature, we do not expect the assumption of stochastically identical sellers to be crucial in determining the results. The analytical results we obtain have to be interpreted more as theoretical insights into key procurement market phenomena than as formulae for practical decision-making.

*Extensions*

Unequal splits are likely to have strategic value for the buyer if they help competition subsequent to the award of the contract. Multiple sourcing can create incentives for sellers to control post-award prices by making two or more committed suppliers compete with each other for larger portions of limited business. An extension of our analysis to a multi-period model (as discussed earlier) that considers post-award controls and risk sharing promises to be a fruitful area for research. Another issue of interest is how to run an optimal multiple source competition. For example, the buyer has a choice of actions if the number of bidders is less than the number of required suppliers. The award may then be made on bases other than price alone (reputation, likelihood of satisfactory supply and the like). These and other considerations should provide fruitful topics for further work in this area.[4]

**Acknowledgements.** This work was partially supported by the Institute for the Study of Business Markets and the Center for Research in Conflict and Negotiation at The Pennsylvania State University. We wish to thank the reviewers, the Area Editor, and the Editor for their constructive comments on previous drafts of this manuscript.

[4] This paper was received in August 1989 and has been with the authors 5 months for 2 revisions. This paper was processed by Marcel Corstjens.

### Appendix: Existence of a Solution to Equation (9)

The expected profit prior to information on the number of entrants may either be higher than the largest feasible value of the opportunity cost, or may lie within the range of the possible values. If the former case obtains, then all firms enter the competition, and $E[\pi_K(n)] = \pi_K(B)$. This is not an interesting case, especially since it implies that the procurement agency is allowing the business to be unnecessarily profitable. If the latter case obtains, then we may denote the probability of entry as $G(\pi_K) = p$, where $G(r)$ is the distribution function of the opportunity cost, $R$. Then $\pi_K = G^{-1}(p)$. $\pi_K$ is a monotonically increasing function of $p$ $(0 \le p \le 1)$. This is evident since $G(\cdot)$ is the cumulative distribution function.

We examine the stochastic dominance property of the binomial distribution to obtain the behavior of $E[\pi_K(n)]$. Define the difference

$$a(n) = \binom{B}{n}[p_1^n(1-p_1)^{B-n} - p_2^n(1-p_2)^{B-n}], \qquad n = 0, 1, \ldots, B.$$

For $p_2 > p_1$, and $(1-p_2) < (1-p_1)$, $a(n)$ is positive for small values of $n$, and negative as $n$ approaches $B$. The difference in the cumulative distributions of the binomial random variable $n$ can be written as

$$C(n^*) = \text{Prob}(n \le n^* \mid p_1) - \text{Prob}(n \le n^* \mid p_2) = \sum_{n=0}^{n^*} a(n).$$

This difference $C(n^*)$ is positive for $n^* = 0$ and is zero for $n^* = B$ (remembering that $\sum_{n=0}^{B} \binom{B}{N} p^n(1-p)^{B-n} = 1, 0 \le p \le 1$).

For values of $n^*$ between zero and $B$, the difference $C(n^*)$ in the cumulative distribution is positive. The difference is depicted as the vertical displacement between the curves in Figure A1.

We conclude that the distribution for $p_2$ dominates that for $p_1$ in the *first-order stochastic* sense, whenever $p_2 > p_1$.

The probability of higher values of $n$ therefore increases as $p$ rises. If $\pi_K(n)$ is a decreasing function of $n$, the expected value of $\pi_K(n)$ with respect to the distribution of $n$ will decrease with a rise in $p$.

If we know that $\pi_K(n)$ is a decreasing function of $n$, then

$$E[\pi(n)] = \sum_{n=1}^{B} \pi_K(n)\binom{B}{n}p^n(1-p)^{B-n} = \pi_K$$

is a decreasing function of $p$.

We have already seen that $\pi_K = G^{-1}(p)$ is an increasing function of $p$. A value of $p$ exists at the intersection of the increasing and decreasing function where the equality holds. A solution therefore exists for the entry condition equation for $0 < p < 1$. The solution is unique since a strictly increasing and a nonincreasing function can have at most one point in common. This is shown as follows:

Let $f(x)$ be strictly increasing and $g(x)$ nonincreasing. Both are assumed everywhere differentiable.

First, $f(x)$ is strictly increasing implies $f'(x) > 0$.

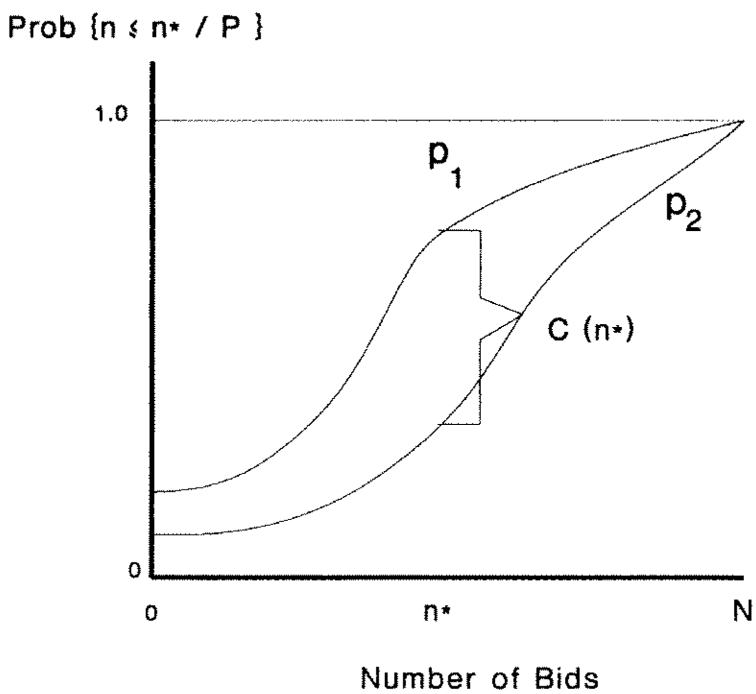


FIGURE A1. Stochastic Dominance.

$C(n^*)$ is the difference between the cumulative binomial probability distributions (continuous approximation shown) of the number of bidders, $n$, when $p_1$ and $p_2$ are the bidding probabilities and $p_2 > p_1$.

Next, $g(x)$ nonincreasing implies $g'(x) \leq 0$.
Thus,

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x) > 0.$$

Let $W(x) = f(x) - g(x)$.
If $W(x_0) = 0$, the solution is unique since

$$W'(x) > 0 \Rightarrow W(x) > W(x_0) \quad \text{for} \quad x > x_0, \quad \text{and}$$
$$W(x) < W(x_0) \quad \text{for} \quad x < x_0.$$

There is at most one solution for $f$, $g$ as above. Q.E.D.

## References


Anton, J. J. and D. A. Yao (1987), "Split Awards, Procurement, and Innovation," Discussion Paper #202, Center for the Study of Organizational Innovation, University of Pennsylvania, Philadelphia, PA.

Boger, D. C. and S. S. Liao (1988), "Dual Awards and Contractor Gaming in Defense Procurement," Competition and Cost Analyses Session, TIMS/ORSA, Washington, D.C.

Canes, M. (1975), "The Simple Economics of Incentive Contracting: Note," *American Economic Review*, 65, 478–483.

Capen, E. C., R. V. Clapp and W. M. Campbell (1971), "Competitive Bidding in High-Risk Situations," *Journal of Petroleum Technology*, 23, 6 (June), 641–653.

Englebrecht-Wiggans, R. (1980), "Auctions and Bidding Models: A Survey," *Management Science*, 26, 2, 119–142.

——— (1987), "On Optimal Reservation Prices in Auctions," *Management Science*, 33, 6, 763–770.

Greer, W. R. and S. S. Liao (1986), "An Analysis of Risk and Return in the Defense Market: Its Impact on Weapon System Competition," *Management Science*, 32, 10, 1259–1273.

Hansen, R. G. (1988), "Auctions with Endogenous Quantity," *RAND Journal of Economics*, 19, 3, 44–58.

Harsanyi, J. C. (1967, 1968a, 1968b), "Games with Incomplete Information Played by Bayesian Players. Part I: The Basic Model," *Management Science*, 14, 159–182; Part II, *Management Science*, 14, 320–334; Part III, *Management Science*, 14, 486–502.

Harstad, R. M. (1990), "Alternative Common-Value Auction Procedures: Revenue Comparisons with Free Entry," *Journal of Political Economy*, 98, 421–429.

Maskin, E. and J. Riley (1984), "Optimal Auctions with Risk Averse Buyers," *Econometrica*, 52, 6, 1473–1528.

McAfee, R. P. and J. McMillan (1986), "Bidding for Contracts: A Principal-Agent Analysis," *RAND Journal of Economics*, 17, 3, 326–338.

——— (1987a), "Auctions," *Journal of Economic Literature*, 25, 699–738.

——— (1987b), "Auctions with a Stochastic Number of Bidders," *Journal of Economic Theory*, 43, 1–19.

——— (1987c), "Auctions with Entry," *Economics Letters*, 23, 343–347.

McCall, J. J. (1970), "The Simple Economics of Incentive Contracting," *American Economic Review*, 60, 837–884.

Milgrom, P. R. and R. J. Weber (1982), "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, 1089–1122.

Ortega-Reichert, A. (1968), "Models for Competitive Bidding under Uncertainty," Technical Report No. 8, Department of Operations Research, Stanford University.

Raffel, D. E. and K. Chatterjee (1989), "Dual Sourcing, Learning, and Constraints on Borrowing," Working Paper, Pennsylvania State University.

Rogerson, W. P. (1988), "Profit Regulation of Defense Contractors and Prizes for Innovation," Working Paper, CMSEMS, Northwestern University.

Rothkopf, M. H. (1983), "Bidding Theory: The Phenomena to be Modeled," in R. Englebrecht-Wiggans, M. Shubik, and R. M. Stark (Eds.), *Auctions and Contracting: Uses and Theory*, New York: New York University Press.

———, T. J. Teisberg and E. P. Kahn (1990), "Why Are Vickrey Auctions Rare?" *Journal of Political Economy*, 98, 94–109.

Samuelson, W. F. (1985), "Competitive Bidding with Entry Costs," *Economics Letters*, 17, 53–57.

——— (1986), "Bidding for Contracts," *Management Science*, 32, 12, 1533–1550.

Seshadri, S. (1988), "Implementing Procurement Objectives in Business Markets," Doctoral Dissertation, The Pennsylvania State University.

——— (1990), "Prices and Prizes: Selection and Control in Multiple Source Procurements," manuscript.

Stark, R. M. and M. H. Rothkopf (1979), "Competitive Bidding: A Comprehensive Bibliography," *Operations Research*, 27, 364–390.

Tirole, J. (1988), *The Theory of Industrial Organization*, Cambridge, MA: The MIT Press.

Vickrey, W. (1961), "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance*, 16, 8–37.

——— (1962), "Auctions and Bidding Games," *Recent Advances in Game Theory*, Princeton University Conference, 15–17.

Weber, R. J. (1983), "Multiple-Object Auctions," in R. Engelbrecht-Wiggans, M. Shubik and R. M. Stark (Eds.), *Auctions, Bidding, and Contracting: Uses and Theory*, New York: New York University Press.

Webster, F. E., Jr. (1984), *Industrial Marketing Strategy*, (2nd Ed.), New York: John Wiley & Sons, Inc.

Wilson, R. B. (1977), "The Loss in Revenue from a Share Auction," Research Paper No. 388, Graduate School of Business, Stanford University.

——— (1982), "Double Auctions," in T. Groves, R. Radner and S. R. Reiter (Eds.), *Information, Incentives and Economic Mechanisms: Essays in Honour of Leonid Hurwicz*, Minneapolis: University of Minnesota Press.