

A MODEL FOR MANPOWER MANAGEMENT*

GARY L. LILIEN† AND AMBAR G. RAO‡

A model for manpower management is presented based upon the study of a large technical department of a corporation. The model can be used in an optimization mode to determine recruiting plans, given manning needs, and in a simulation mode to project the results of changes in salary structure or advancement policies. The model explicitly recognizes different levels of performance at each job level, and the fact that certain sources of employees are more likely to produce "better" employees than others. Also modelled is the phenomenon of the "learning period"—the time it takes for a person to approach his potential in a given job. Several numerical examples of the use of the model based on observed data are presented.

Introduction

Manpower planning has attracted the attention of management scientists in the past few years as a fruitful area both for theoretical analysis and practical application. There are three main directions in which research efforts have been channeled. The first is the application of techniques of mathematical programming to determine hiring and training requirements. Usually historical data are used to determine a matrix whose entries correspond to probabilities of movement from one state (e.g. job level) to another within the system and to probabilities of leaving the system from given states. For instance, see Charnes, Cooper, Niehaus and Sholtz [2], Charnes et al. [3] and Patz [9]. A second direction of research has been a stochastic process approach seeking to model the hiring, advancement and promotions of individuals in an organization as a birth and death process. Using this approach the probability distributions of various system statistics can be obtained (see Dill, Gaver and Weber [4] and Vroom and MacCrimmon [10] for examples of this approach). A third and potentially fruitful avenue of research has been to explain turnover behavior as a function of various personal, professional and organizational factors. Examples of this type of work are Galbraith [7] and Farris [5]. Two nontechnical surveys of manpower planning are Walker [11] and Ferguson [6].

The authors were involved in a study aimed at the development of rational manpower policies in a large corporation. The goals of the study included the specification of relevant information for decision making as well as modelling the hiring and promotion decisions per se. A large technical department in the corporation was selected for a pilot study. This department had suffered from a high turnover rate and adequately qualified personnel were very hard to find. Management projected an increasing demand for the services offered by the department, making the problems particularly acute. Discussions with departmental managers and with personnel people indicated that several questions had been raised (and gone unanswered) regarding alternative hiring and promotional practices within the department. For instance, it was not known whether employees recruited from certain sources performed sig-

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† Massachusetts Institute of Technology.

‡ New York University.

nificantly better than from other sources; nor did management have a good estimate of the probable length of time a particular type of employee would remain in the department. In the area of salary administration, an important question was whether or not a higher salary structure would result in a significant decrease in turnover, and more importantly, what decrease in turnover rate would justify (financially) a given increase in salary. The following sections present the preliminary data analysis, the mathematical model that was developed and numerical examples of the use of the model under different planning policy assumptions.

While the model developed is essentially a linear programming model, it differs in several important ways from other models using similar techniques. First, it explicitly recognizes the different levels of performance by people in the same job classification, and their differential rate of movement through (and out of) the system. Secondly, the model allows for the fact that there is a "learning period" after a person moves into a new job and before his maximum potential in that job is attained. Third, the value of an individual to the organization is incorporated into the objective function of the model. This permits the investigation of the impact of alternative salary structures and promotion rates on the profitability of the organization as a whole. In conducting such sensitivity studies, managerial judgement is combined with estimates from historical data to parameterize the model. The authors feel that this is a most valuable use of a model quite apart from the ability to generate optimal solutions (see Little [8] for a discussion of this viewpoint).

Preliminary Data Analysis

The department selected for the pilot study maintained fairly detailed records on all its present employees and many who had left in the recent past. These records formed the data base for the study. The records gave salary histories, descriptions and dates of previous jobs, demographic and educational details, etc. In addition, assessments of employee performance which were made periodically were available.

The records were analyzed in several ways, with the objective of discerning whether certain types of employees performed significantly better than others, and whether or not certain types were likely to stay longer than others. It was found that demographic and socioeconomic characteristics had little predictive power with regard to performance; it was found, however, that employees with larger families, although poorer performers, tended to stay longer than those with smaller families. It was not clear, though, how this fact could be used in determining hiring policies. It was more fruitful to relate performance to the previous jobs held, and also to relate performance to length of stay with the company. Significant differences were found using these criteria—that is, certain sources seemed to produce a higher proportion of superior employees than others. Also, on the average these superior employees stayed with the company a shorter time than other employees. This last discovery was not really surprising in view of the demand for highly skilled people in the particular field. Many of these observations are consistent with the findings of Farris [5].

The data analysis led to the development of statistics relating to the performance that could be expected of various types of employees, and to statistics estimating the likelihood of the different employee-types staying in various jobs for given lengths of time. These statistics together with various cost elements were then combined into a formal manpower planning model, described in the next section.

Mathematical Model

Most previously published work treats all prospective employees as if they were indistinguishable. As mentioned above, one of the most important discriminating characteristics between "better" and "poorer" performance by employees is the source from which these employees were obtained. Assume that a company can distinguish between its better and its poorer employees (something all employers do in practice) and also that it has historical data on the performance of its employees with respect to the source from which the employee was obtained. Under these conditions the decision problem of how many employees to hire in each time period from each of a number of sources is formulated below. A self-contained unit of a large corporation is considered here.

Transitions outside the unit will be considered indistinguishable from an employee's leaving the company—optimization will be from the subunit standpoint. A model of the following sort for an entire company would be too large to be feasible—if the company performs the calculation for a group of subunits, then an incremental cost analysis can give a good plan for the entire company.

Let employee type 1 be "better" performers, employee type 2 be "poorer" performers, and assume that there are a number of job levels $1, \dots, h, \dots, H$. The possible movements that an employee can make from one time period to the next can be represented as follows.

An employee of performance type 1, job level h , who has been in his job for i time periods can (1) leave the unit, (2) be in the same job and have the same rating in the $i + 1$ st period, (3) be in the same job but with a performance rating of type 2 in the $i + 1$ st period, (4) be promoted to job level $h + 1$, rating 1, or (5) be promoted to job level $h + 1$, rating 2. For simplicity, promotions of more than one job level and demotions have not been considered. If however, actual employee data indicate that positive transition probabilities are associated with either of these events, then they can be easily be included. Let

$P11_{i,i+1,h}$ = Probability that an employee of rating type 1, job level h , having been in his job for i time periods since his last promotion, does not get a promotion, stays with the company and remains an employee of type 1. Similar definitions can be constructed for $P12_{i,i+1,h}, P21_{i,i+1,h}, P22_{i,i+1,h}$.

Let node $N + 1$ refer to "out". This implies that after some number of time periods, N , an employee's set of transition probabilities stabilizes, i.e. period $N + K$ will be indistinguishable from N ($K = 1, 2, \dots$) for purpose of analysis. These transition probabilities are assumed to be stationary: stationarity properties are not critical to this analysis, however, so if the transition probabilities can be estimated in a time-variable manner then they can be used as such. To continue, let

$P11_{i,N+1,h}$ = Probability that an employee of type 1, job level h , in a job for i time periods, leaves the system (quits or gets transferred). Similarly for $P22_{i,N+1,h}, P12_{i,N+1,h}, P21_{i,N+1,h}$, are defined = 0.
 $P11_{i,1,h}$ = Probability that an employee of type 1, job level h , in job for i time periods, is promoted and remains an employee of type 1 after he is promoted. Similarly for $P12_{i,1,h}, P21_{i,1,h}, P22_{i,1,h}$.

Suppose management wishes to maintain the manpower levels at $L_{t,h}$ employees during time periods $t = 1, \dots, T$, job levels $h = 1, \dots, H$. Suppose, also there are K hiring sources (or sets of sources) noted as $k = 1, \dots, K$. Then let:

- $n_{k,t,h}$ = The number of employees hired at time t (to start work at time $t + 1$) from source k for job level h .
- $d_{k,h}$ = Probability that a man hired from source k for job level h obtains a rating of type 1 in his first employment period. A "source" here can be, for example, (1) a particular university (or group of universities), (2) another company (or group), (3) other departments (or groups) within the same company, (4) a particular agency, etc.
- $1 - d_{k,h}$ = Probability that a man hired from source k for job level h obtains a type 2 rating in his first employment period.
- $m1_{i,t,h}$ = Number of employees of type 1 rating in job h at time t who have been in their present position for i time periods. Similarly for $m2_{i,t,h}$.
- $m1_{i,0,h}m2_{i,0,h}$ = Initial values of the current employee distribution.
- $r1_{i,h}$ = Revenue (or cost) associated with node $(1, i, h)$.

The problem of assigning a value to an employee who performs at a certain level has been approached as follows: recognizing that an employee's value to a company is related to his ease of replacement and, hence, to his cost to the company (his salary + overhead), one can assume his value to be *determined by the market* as a function of his salary. In other words, his salary and overhead can be treated as an investment generating the company's rate of return.

In addition, we must recognize that it takes some time for an employee in a new job to reach his full potential, and further that in the early part of his tenure in this job his value may actually be negative. A specific relationship is considered in the numerical example presented later in this paper. Let

$C_k(n_{k,h})$ = Cost associated with obtaining n employees at level h from source k .

In general, this function will be composed of a setup cost plus a cost related to the number hired. The cost related to the number hired was found to be approximately linear in some region before rising at an increasing rate. If one recognizes the normal policy of large companies to maintain the elements of the setup cost component even when not actively recruiting at a source in order to maintain corporate image and to advertise, then the setup cost can be eliminated from the analysis as a fixed cost. If, in addition, operation is permitted only in the area of linear cost, ensured by placing an upper bound on each $n_{k,t,h}$, the cost function will be linear: $C_k(n_{k,t,h}) = C_k n_{k,t,h}$.

The problem of selecting hiring sources to fill personnel requirements can now be formulated as follows. Find the set of $\{n_{k,t,h}\}$ to

$$\max Z = \sum_{t=1}^T \left(\left[\sum_{h=1}^N \sum_{i=1}^{N+1} \sum_{j=1}^{N+1} m1_{i,t-1,h} (P11_{i,j,h} r1_{j,h} + P12_{i,j,h} r2_{j,h}) \right. \right. \\ \left. \left. + m2_{i,t-1,h} (P21_{i,j,h} r1_{j,h} + P22_{i,j,h} r2_{j,h}) \right] - \sum_{k=1}^K C_k n_{k,t-1,h} \right) [1/(1+R)]^t,$$

subject to the following constraints.

(1) Maximum hiring quantity constraint:

$$n_{k,t,h} \leq l_{k,h} \quad \text{for all } t, k, h.$$

(2) Satisfaction of employee requirements constraint:

$$L_{t,h} - \Delta_L \leq \sum_{i=1}^N m1_{i,t,h} + m2_{i,t,h} \leq L_{t,h} + \Delta_u \quad \text{for all } t, h.$$

(3) Employee advancement constraints:

$$(a) \quad m1_{i,t,h} = m1_{i-1,t-1,h}P11_{i-1,i,h} + m2_{i-1,t-1,h}P21_{i-1,i,h}, \quad N \geq i \geq 1,$$

for all t, h similarly for $m2_{i,t,h}$;

$$(b) \quad m1_{N+1,t,h} = \sum_{i=1}^N m1_{i,t-1,h}P11_{i,N+1,h} \quad \text{similarly for } m2_{N+1,t,h};$$

$$(c) \quad m1_{1,t,h+1} = \sum_{i=1}^N m1_{i,t-1,h}P11_{i,1,h} + m2_{i,t-1,h}P21_{i,1,h} \\ + \sum_{k=1}^K n_{k,t-1,h}d_k \quad \text{for all } t, h = 0, \dots, N - 1.$$

For $h = 0$ above, $P11_{i,1,h} = P21_{i,1,h} = 0$. Similarly for $m2_{1,t,h+1}$. If constraints for employee mix are desired, they may be added as:

$$(4) \quad CU_{i,t,h} \leq \sum_{i=1}^N m1_{i,t,h} \leq CL_{i,t,h} \quad \text{for all } t, h$$

where $CU_{i,t,h}$, $CL_{i,t,h}$ are upper and lower bounds, respectively.

The problem formulated above is an integer linear program which can be solved in theory at least. If the particular problem as formulated above cannot be solved in a reasonable time by existing integer codes, then two choices are open.

- 1) The state space (and, thus, solution space) can be collapsed to permit solution in reasonable time. With a suitable understanding of the problem structure this can be done in many cases with little loss.
- 2) Relax the constraint on an integer solution. This will yield nonoptimal solutions after rounding, but the problem formulation is already a broad generalization of the real situation. One should not expect that the hiring policy which this solution strategy indicates is to be used as such. Rather, the results should be used to indicate the relative merits of hiring employees from various sources in a rather general way. The results should indicate the directions for desirable changes, but not necessarily the precise magnitude of the changes. Thus, the removal of the restriction to integer value does not lessen the value of the solution significantly.

In such time-staged linear programming models, decisions to be undertaken towards the end of the planning horizon are often affected by the finite nature of problem formulation. One way of avoiding this "end error" is by associating a "salvage value" with employees at the end of the horizon. In practice, however, only the first year's decisions are likely to be implemented before the model is re-run. Hence the decisions towards the end of the planning horizon indicated by the model have little practical value and in a well-formulated model have negligible effect on the initial decisions. Thus, provided a planning model is updated frequently, errors induced by the assumption of a finite horizon are likely to be small.

Example

The structure of the example is based on data from a large technical department in a major industrial corporation. The data have been altered for proprietary reasons but the problem-structure is still indicative of the real-life situation.

Our example considers a department with three job-categories (levels) '1', '2', and '3' with level 3 being the highest. After 4 years in the job an employee is assumed

to have reached a "steady-state" there—in other words, employee states will be '1', '2', '3', and '4 or more' years in the job for each job level. In addition, we consider two performance levels or "types" of employees: type '1', superior, type '2', inferior. We also consider an absorbing state with no revenue generation called "OUT", representing the situation when an employee leaves the system. Thus, the model has $1 + 2 \times 3 \times 4$ or 25 states.

Next we record the historical employee transitions within the department. The yearly transition rates were tested for stationarity and agreed with the markovian hypothesis before aggregation (see Anderson and Goodman [1] for details). Table 1 gives the number of observed state-to-state transitions and associated row sums for an equivalent of four years of observations. The transition numbers divided by the row totals give the estimates of transition probabilities used in the model. For example, the probability that a type 1 employee in job level 1 for 2 years will also be a type 1 employee in job level 1 next year (i.e. TIME = 3) is $22/51 = 0.43$. (These are maximum likelihood estimates for the transition probabilities.)

Also included in Table 1 is the initial employee distribution vector. This implies that there are currently 16 type 1 employees in job level 1 who have been there less than 1 year. This transition matrix and initial employee distribution will be the basis of the computational structure of the model.

The employee-value functions have been chosen to be of the following form:

$$\text{Value} = \{K_0[1 - K_1 \exp(-(T_1 - 1)K_2)] - K_3\}[1/(1 + R)]^T,$$

where

- R = Corporate rate of return,
- T_1 = Number years employee is in job,
- T = Year index,
- K_0 = The employee's ultimate worth,
- K_1 = Learning potential of employee. If $K_1 = 0.4$, employee's initial value is 60% $(1 - 0.4)$ of his ultimate value,
- K_2 = Learning constant, the larger K_2 , the faster an employee's initial value reaches his ultimate value. We set $K_2 = 0.69$. This combined with the value selected for K_1 implies that an employee reaches 90% of his potential value at $T_1 = 3.0$.
- K_3 = Employee's cost to the company (salary + overhead).

Table 2 gives the values of these constants used in this example:

We assume that for each job level there are two sources (or groups of sources) from which to recruit employees: A "Rich" source—more expensive per employee recruited and with a higher probability of that employee being a type 1 employee in his first year in the job, and a "Poor" source, less expensive and lower probability of type 1 employee. In addition, for each source there are annual recruitment limits on the sources (which will be assumed here to be constant year-to-year). These details are shown in Table 3.

We must now set limits on the number of employees of each type needed in the system. (These numbers are usually made available through familiar departmental "Five Year Plans".) Assume a limit of 230 level 1 employees, 95 level 2 employees, and 40 level 3 employees in the usual pyramid-like structure. Assume that a lower limit of 20 and an upper limit of 85 are set for the number of type 1 employees in the

TABLE 2
Employee Value Parameters

Job Level	Performance Type	K_0	K_1	K_2	K_3	R
1	2	10,000	0.4	0.69	8500	0.08
	1	15,000	0.4	0.69	9200	0.08
2	2	15,000	0.4	0.69	12750	0.08
	1	22,500	0.4	0.69	13800	0.08
3	2	20,000	0.4	0.69	17000	0.08
	1	30,000	0.4	0.69	18400	0.08

system each year. This kind of restriction is often imposed by management in the belief that a minimum number of good people is necessary, but too many is undesirable.

The cost studies discussed here will be run for a planning horizon of ten years. The combination of this planning horizon and the 8% discount rate is not enough to strictly eliminate the end-off effect or salvage value problem for employees hired late in the planning horizon. However, for simplicity we have chosen to ignore the problem here, for the reasons given earlier.

Some important simplifications have been made here. We have structured a problem in which optimization takes place from the subunit standpoint. We could have gotten around that by associating an expected value-return from an employee leaving the department but remaining in the company. Mix constraints and employee requirements could have varied from year to year, and other constraints and complications could have been added. This example, however, retains sufficient complexity to illustrate the power of this modelling approach.

The linear program outlined above translates into a 170 row, 60 variable problem and was solved in 11 seconds on an IBM model 360-75 using MPSX. Table 4 gives some details of the solution.

(The unusual looking hiring pattern from source 1 occurs because recruiting from that source in year 2—more “promotable” employees on the average—would cause a violation of level 2 employee constraints in years 3 and 4.)

Analysis of the dual solution indicates where the strongest constraints are. Some of these are listed in Table 5.

Here, for example, it is worth \$1337 to hire one extra employee from source 1 in year 1. And the value to the organization is \$5200 for relaxing the constraint on the number of level 3 employees in the organization in year 1.

The objective function value of \$1.3 million is not as meaningful as the direction and proportion of change in the objective function when characteristics of the model

TABLE 3
Employee Recruitment Parameters

Job Level	Source	Prob(Type 1)	Cost/Employee	Recruitment Limit
1	“Rich” (R)	0.7	\$3000	25
	“Poor” (P)	0.2	1500	Unlimited (UNL)
2	R	0.5	3500	6
	P	0.15	2500	UNL
3	R	0.5	3500	3
	P	0.15	2500	UNL

TABLE 4
Hiring Requirements—Optimal Policy

Year	Source					
	1	2	3	4	5	6
1	25	47	6	3	3	10
2	0	65	0	0	3	8
3	25	8	0	0	3	5
4	25	9	0	0	3	5
5	25	12	0	0	3	3
6	25	13	0	0	1	0
7	25	11	0	0	0	0
8	25	0	0	0	2	0
9	25	0	0	0	3	0
10	19	0	0	0	0	0

structure are modified. We consider two such modifications here: a change in turnover rate and a change in the mean amount of time employees spend in their jobs. The purpose of such sensitivity studies is to infer the magnitude of the value to the firm of policy changes (salary and benefit structure changes, promotional policy changes, etc.) which are expected to yield changes in the structure of the system.

To explore the effect of reducing turnover, assume that an $X\%$ increase in turnover leads to decreases in the other transition probabilities proportional to the current transition probabilities. For example, the current turnover probability for level 1, type 1 employees at time 1 is $13/90$ or 0.144 . A 10% increase in this turnover makes it 0.159 (an increase of 0.015) and, by our assumption, the new transition probability to type 1, level 1 at time 2 is $(44/90) - (44/(90-13)) \times 0.015$. Several cases were run under this assumption; the results are presented in Figure 1.

The results from these runs may be counter-intuitive; here an increase in turnover leads to an increase in value to the organization. This might be explained as resulting from the pressure of type 1 employees to become type 2 employees over time—it seems to be more valuable to keep turnover higher, replacing those who leave from the "Rich" sources if possible and keeping a higher average quality of employee than to keep employees in the organization for a long time.

Another study that management might want to make is of the following sort: most hiring occurs in level 1 which seems well able to support level 2 requirements in most

TABLE 5
Constraint Values

Constraint	Dual Activity
1. Source 1 hiring, Year 1	\$1337
2. Source 2 hiring, Year 1	1563
3. Source 3 hiring, Year 1	2142
4. Level 1 Employees, Year 1	657
5. Level 2 Employees, Year 1	3970
6. Level 3 Employees, Year 1	5200
7. Employee Mix Constraints—All Years	0

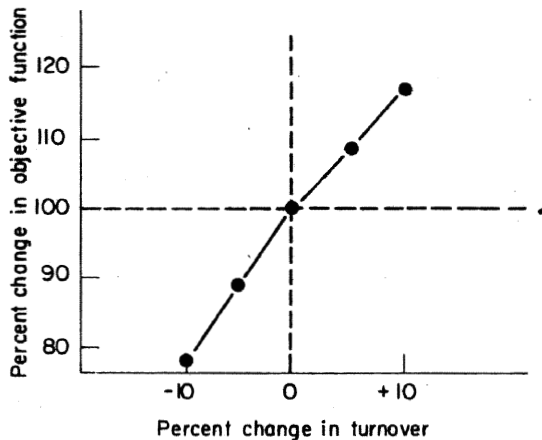


FIG. 1.

FIGURE 1. Percent Change in Turnover.

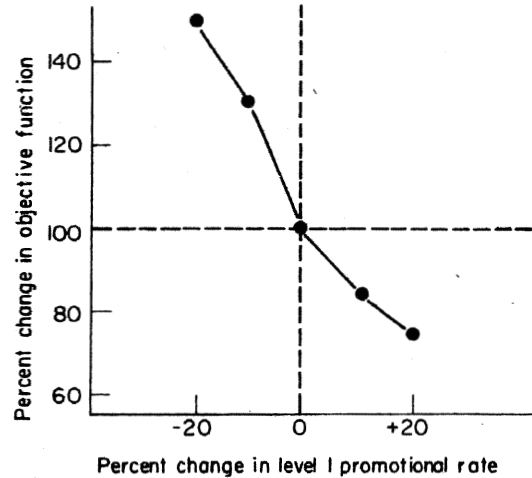


FIG. 2.

FIGURE 2. Percent Change in Level I Promotional Rate.

years through promotions. What, then, is the value of an $X\%$ decrease in promotional rate for level 1 employees, assuming that change is distributed proportionally across the other probabilities as before?

Such a set of runs was made and the results are plotted in Figure 2. Here the results are more consistent with intuition: decreases in promotional rates yielded increases in value. A 10% decrease raised the objective function 31%.

Summary and Conclusions

A model for manpower management, based on the authors' experience with a large technical department of a corporation, was developed. The model explicitly considered the fact that managers classify employees into good and poor performers, that certain sources of new employees are more likely to produce good performers than others and that there is a period of learning before a person reaches his full potential in a job. Numerical examples of the use of the model were presented, first to determine the pattern of recruitment from various sources, given manning requirements, that maximizes a measure of performance of the department. Next some of the parameters of the model were varied to determine the effect of changes in turnover rate and rate of promotion to higher job levels.

The model presented is descriptive of the movement of individuals through an organization. It does not attempt to explain *why* people leave an organization or why they join it in the first place. We believe that the development of such explanatory models is likely to be a fruitful area for future research.

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