A GAME-THEORETIC ANALYSIS OF CAPACITY COMPETITION IN NON-DIFFERENTIATED OLIGOPOLISTIC MARKETS[1]

James A. Dearden, Gary L. Lilien and Eunsang Yoon

Lehigh University, USA

The Pennsylvania State University, USA

University of Massachusetts at Lowell, USA

Abstract

High capital investment industries often see regular cycles of over capacity followed by under capacity. We develop a game theoretic model and show that such cyclical behavior can exist in equilibrium, even if demand and prices are stable and if firms consider the capacity strategies of other firms. We discuss the implications of these findings for individual firm strategies that might reduce the impact of those cycles and for regulatory or industrial policies that might lead to more efficient market operations.

1. Introduction

The dynamics of high capital investment markets produce cycles of various sorts. Those cycles are highlighted in the business press, in numerous academic studies and in the everyday discussions with practicing managers. Consider the following:

> "Dennis H. Reilly of DuPont, speaking at a December 1993 briefing in London, pointed out that the major problem for the [Titanium Dioxide--TiO_2] industry is massive global overcapacity. ...Prices are at too low a level to justify investment, said Reilly. Although new investment is not needed now, when the recession ends and growth for TiO_2 picks up, there will be a shortage." (*Chemical & Engineering News*, January 3, 1994, p. 14.)

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The dynamics of the TiO_2 industry are far from uncommon. The capacity cycle problem is summed up by:

> "In the familiar boom-bust pattern of the not-too-distant past, managers added production capacity, allowed overhead to swell, and stockpiled inventories in antipication of rising demand during expansions. When the economy tanked, they shut factories, laid off workers, iced new-product development, and purged excess inventories at distress prices." (*Fortune*, August 7, 1995, pp. 59-60.)

What is happening here? It appears that firms, privy to similar but noisy and often confusing information about what short-term and long-term demand for the output of their industry is or is likely to be, commit capital to add capacity or selectively delete capacity. In capital intensive industries, low capacity utilization adds a substantial cost burden to each unit sold; high capacity utilization leads to low unit costs and higher profits, as well as the tantalizing prospect of adding to firm profits by expanding.

In a monopoly, a firm might tune its production capacity to track demand cycles so that its capacity utilization was optimal in some long run profit maximizing sense, perhaps. Most oligopolies preclude such firm policies, however.

Is the phenomenon real or imagined? Consider Exhibits 1 to 3, based on aggregate US statistics on capacity utilization published every month by the Federal Reserve. The data show clear and persistent cycles in capacity utilization over the past several decades in the US. As the industry definition gets narrower, these cycles typically get more severe. Exhibit 1 shows that the capacity utilization of all US manufacturing, mining, and utilities combined has fluctuated from the low 70%'s to 90% over the last 30 years. As Exhibit 2 and 3 illustrate, similar patterns exist in individual manufacturing industries, but they are generally more severe and erratic, with fluctuations ranging from the low 40%'s to peaks of nearly 110%.

Are these just natural business cycles that promote a competitive marketplace? We think not: "...the structure of an industry may be so dysfunctional to the results of competition that collective action is appropriate to fix it. In such an instance the workings of the market produce neither efficiency nor profit" (Bower, 1986, p 14). And these fluctuations are not good for customers either, who see widely varying levels of supply assurance and prices. While these cycles may result from strategic competition for market leadership, they are dreaded by buyers, sellers and government regulators alike. Hence this paper will investigate some possible causes for these cycles and speculate what actions firms and regulators might take to address the situation.

Exhibit 1. Capacity utilization for total industry, 1967-1995

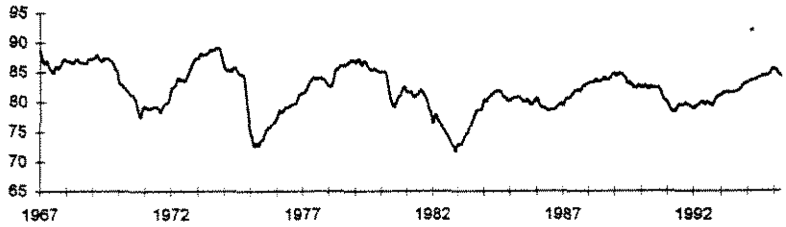


Exhibit 2. Capacity utilization for iron and steel, 1967-1995

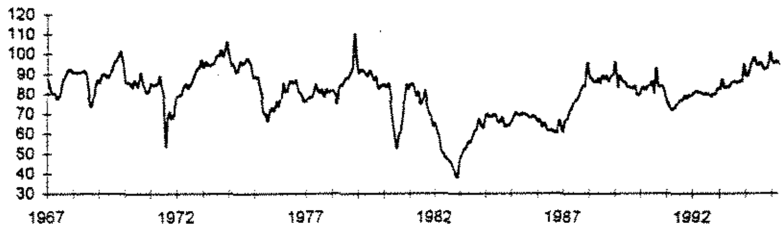
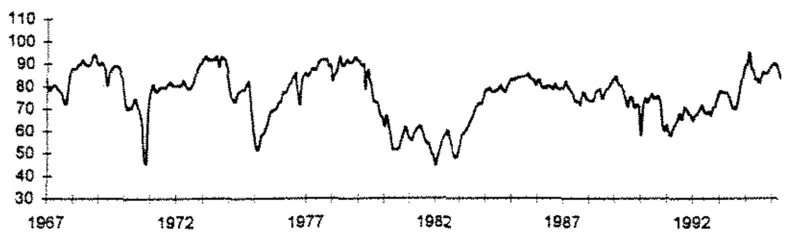


Exhibit 3. Capacity utilization for motor vehicles and parts, 1967-1995



To simplify our analysis and make it more specific, consider mature, non-differentiated oligopolistic industries. In such industries price is likely to be determined by the supply-demand market relationship that characterizes commodity or near-commodity markets. Due to the undifferentiated nature of the product, competition in such markets involves perceptions of quality, reliability and assurance of supply, elements that are closely related to a selling firm's share of production capacity. Many chemicals, metallic products, and electrical and electronic components are relevant examples. In such markets production cost is related to capacity utilization and market share is related to capacity share.

In the next section, we briefly review some of the related literature on the topic. Then, we describe the emprical findings in the Dearden, Lilien, and Yoon (1996) empirical analysis of the titanium dioxide (TiO_2) and Zicron industries. (Zicron is a fictitious name to preserve the proprietary nature of the data.) We use these empirical findings to aid in our choice of a game theoretic model to examine capacity cycles. Three findings, in particular, are relevant.

First, capacity cycles are often preceded by a market demand shock. Second, the empirical findings indicate that capacity, and not price, determines market shares. Hence, we choose a model of product-market competition whose equilibrium has this feature. Third, the exploratory analysis suggests that both pre-capacity and post-capacity marginal costs are constant in output.

In the following section we develop a game theoretic model incorporating many of these phenomena. The results of that model suggest that after a demand shock, the lack of capacity change coordination among an industry's firms prompts capacity cycles. We then present evidence that capacity changes in the titanium dioxide industry are consistent with the equilibrium results of the game theoretic model. In the final section, we discuss the implications and limitations of our work.

2. Related Literature

The literature in oligopoly theory related to our research has primarily considered capacity expansion decisions. Friedman (1983, Chapter 7), Gilbert (1986), and Fudenberg and Tirole (1986) give excellent surveys of different aspects of this literature. A more recent examination of capacity and competition is by Gal-Or (1994). The entry model of Dixit and Shapiro (1985) also shares some features of interest with the problem we consider.

The models of capacity expansion in this literature differ in several respects. First, some treat capacity as a physical limit on production, so that capacity expansion relaxes a constraint (for example, Prescott, 1973). Others treat additions to the capital stock as 'deepening' capital (or capacity) by shifting the marginal cost curve downwards (for example, Flaherty, 1980). Friedman (1983, p. 166) describes this a being "nearer to the spirit of neoclassical marginalist economics," and is the tack that we follow. (Note that we use the terms capacity and capital synonymously in this section.)

Second, the literature differs on whether a dominant firm exists. The seminal articles of Spence (1977) and Dixit (1980) and various sequels, such as Fudenberg and Tirole (1983), Ware (1984), and Arvan (1986) assume that there is a dominant firm, which moves first to add capital. Ghemawat (1984) shows that the firm with the greatest installed capacity (by assumption, the one with the lowest cost of additional investment) will add to capacity when only one firm is allowed to do so.

In the absence of 'natural' market leaders (like first entrants in models where firms enter sequentially over time), the presence of dominant firms cannot be assumed but must be endogenous to the analysis. (We will do this later by modifying the extensive form, without imposing the arbitrary constraint that only a single firm can add capacity in any given period.)

Third, the literature differs on whether or not capacity is continuous or "lumpy." We follow Ghemawat (1984) and much of the operations research literature (see, for example, Friedenfelds, 1981) in assuming that additions to capacity are 'lumpy' and that financial constraints may preclude firms from adding more than a given number of lumpy units of capital in a period.

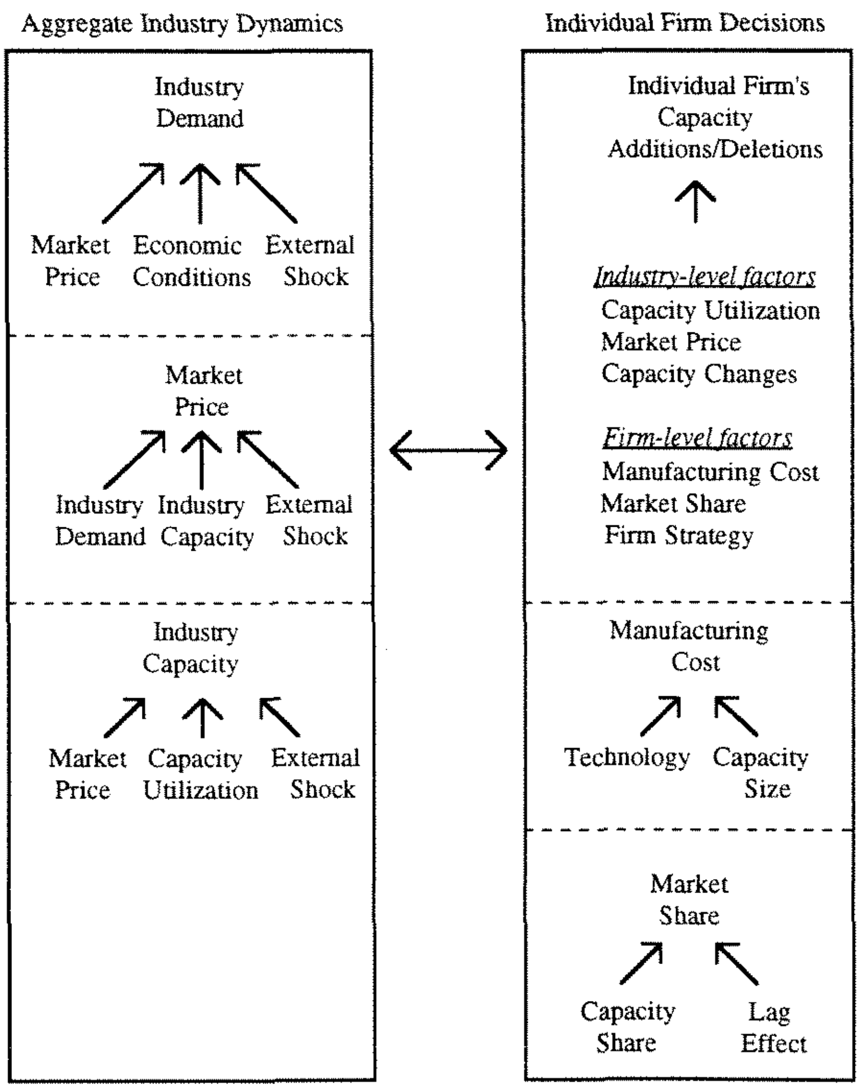
Fourth, the literature differs in the consideration of production and demand dynamics. Dixit (1980), Ware (1984), and Arvan (1986) examine models in which firms compete in an output market in only one period. These models therefore cannot capture the overinvestment-underinvestment cycle that is endemic in many industries. Spence (1977), Friedman (1983, Chapter 7), Flaherty (1980), and Benoit and Krishna (1987), among others, on the other hand, consider dynamic models in which firms add capital and compete in an output market in an infinite number of periods. We will develop a two period model, the minimum number of periods needed to allow cycles to emerge.

3. Capacity Cycles in the TiO_2 and Zicron Industries

Dearden, Lilien, and Yoon (1996) analyze the capacity addition/deletion decisions for the TiO_2 and Zicron industries for the years 1970-1985. Exhibit 4 displays the results of this analysis qualitatively. Theier results generally suggest that overcapacity and undercapacity cycles are likely to occur because firms, acting on market signals,

Exhibit 4

Qualitative Summary of Empirical Results on the Dynamics of Capacity Addition and Deletion Decisions in the Titanium Dioxide and Zicron Industries



Source: Dearden, Lilien, and Yoon (1996) simultaneously add (perhaps too much) capacity in good times, and delete (perhaps too much) capacity in poor times.

The empirical models they report fit and predict aggregate industry dynamics and individual firm decisions well. They found a high degree of sensitivity of a given firm's capacity addition and deletion decisions to other firms' capacity changes and industry capacity utilization. This finding confirms simultaneous, competitive firm behavior. They also found, via discriminant analyses, that a firm's price and its market share may be due to plant-specific factors or strategic differences, issues that we will explore in the next section and discuss later in more detail.

4. A Formal Model of Competitive Capacity Decisions

The empirical analysis in Dearden, Lilien, and Yoon (1996) suggests that firm capacity addition and deletion behavior can be explained by a series of regression models that do not incorporate knowledge or anticipation of competitive actions. Under such circumstances, where firms ignore the strategic actions of others, fluctuations in market demand lead to capacity cycles. But what if demand were stable over time and firms did consider the actions of other firms in their decision processes? Would we still see such cycles?

The results from Dearden, Lilien, and Yoon (1996) and the theoretical literature on capacity expansion (Section II) lead us to consider a model that should admit the following features:

i. As suggested by the preliminary analysis, pre-capacity and post-capacity marginal costs are both constant in output.
ii. Because capacity, and not price, determines market shares, we model the product market competition as Cournot competition.
iii. Capital addition both relaxes a capacity constraint and lowers marginal cost for outputs above the previous capacity output and below the new capacity output.
iv. There is no dominant firm or 'natural' market leader.
v. Capital addition is lumpy.
vi. There are at least two periods of product-market competition and potential capital change.

4.1. Model Formulation

Consider an industry with two firms, indexed as i=1,2. There are three important elements to our model: (i) the cost functions, (ii) the market demand function, and (iii) the timing of the firms' decisions.

The cost structure. Firm i's cost function has two components -- fixed and variable costs -- and both of these components depend on the firm's capital for capacity. Firm i's fixed cost function is $F_{it}(K_{i(t-1)}+d_{it})$; and its variable cost function is $V_{it}(K_{i(t-1)}+d_{it},Q_{it})$; where $K_{i(t-1)}$ is firm i's capital at time t-1, $d_{it} \in \{0,x\}$ is the capital that firm i changes in period t, and Q_{it} is firm i's output at time t. The capital change is constrained to be either 0 or x; that is, investment is lumpy.

We assume that $F_{it}(K_{i0}+x) > F_{it}(K_{i0})$, i.e., that capital addition raises fixed costs. In particular, we examine a specific form of the cost function:

$$F_{it}(K_{it}) = \begin{cases} \underline{F}_i \text{ if } K_{it} = K_{i0} \\ \bar{F}_i \text{ if } K_{it} = K_{i0}+x \end{cases} \quad (1)$$

where $\bar{F}_i > \underline{F}_i$. We also assume that $V_{it}(K_{i0}+x,Q_{it}) \le V_{it}(K_{i0},Q_{it})$; capital addition for capacity does not raise variable costs. In particular, the variable cost function is

$$V_{it}(K_{it},Q_{it}) = \begin{cases} \underline{c}_i Q_{it} \text{ if } K_{it} = K_{i0} \text{ and } Q_{it} \le \hat{Q}_i \\ \underline{c}_i \hat{Q}_{it} + \bar{c}_i(Q_{it} - \hat{Q}_i) \text{ if } K_{it} = K_{i0} \text{ and } Q_{it} > \hat{Q}_i \\ \underline{c}_i Q_{it} \text{ if } K_{it} = K_{i0}+x \end{cases} \quad (2)$$

where $\hat{Q}_i$ denotes firm i's output at capacity, $\underline{c}_i$ denotes firm i's marginal cost for pre-capacity outputs, $Q_i \le \hat{Q}_i$, and $\bar{c}_i$ denotes firm i's marginal cost for for post-capacity outputs (where $\bar{c}_i > \underline{c}_i$). Thus, marginal cost is greater for pre-capacity outputs than for post-capacity outputs. Also, capital addition from K_{i0} to $K_{i0}+x$ increases capacity so that it is no longer binding at all equilibrium output rates. With the capital level $K_{i0}+x$, firm i then produces at marginal cost $\underline{c}_i$.

The market demand function. The firms produce an undifferentiated product, compete in a Cournot market, and face a linear demand curve

$$P_t = a - b(Q_{1t} + Q_{2t}) \tag{3}$$

where P_t denotes the industry price at time t, and $a,b > 0$ are parameters.

The extensive form and timing. We consider a 4-stage game, where t=1,2 correspond to the first period and t=3,4 correspond to period 2. In the first period: in stage 1, the firms simultaneously set capital levels; in stage 2, there is Cournot (quantity) competition. The second period is identical to the first period. Exhibit 5 presents the extensive form of this game.

The profit function. From the cost structure, market demand, and the extensive form, firm i's profit function is

$$\pi_i = [a - b(Q_{i2},Q_{j2})]Q_{i2} - V_{i2}(K_{i2},Q_{i2}) - F_{i2}(K_{i2}) \tag{4}$$

$$+ \delta\ [[a - b(Q_{i4},Q_{j4})]Q_{i4} - V_{i4}(K_{i4},Q_{i4}) - F_{i4}(K_{i4})\ ],$$

where δ = 1/(1-discount rate) denotes the discount factor from period 1 to 2.

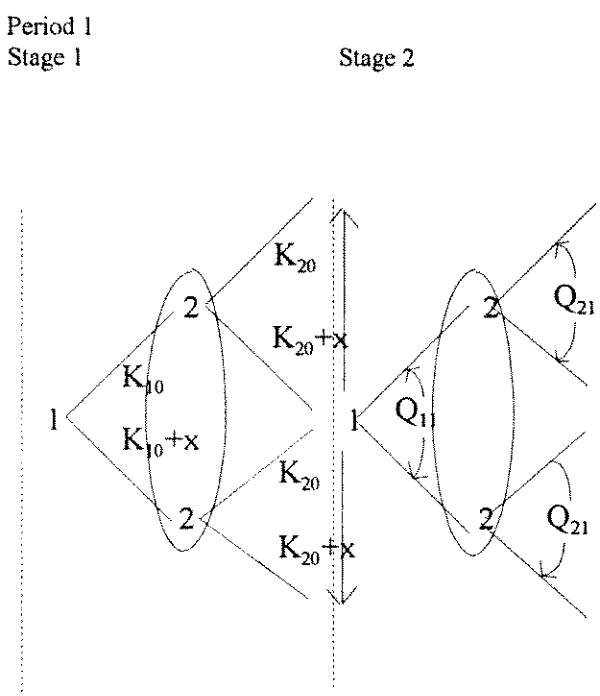
4.3. Model Results

Existence of Equilibrium. Establishing the existence of a subgame perfect equilibrium is straightforward. Given capital stocks and our assumptions about the nature of competition, at stages 2 and 4, there is a well-defined strictly concave profit function for each firm. A unique Nash equilibrium exists for the stages 2 and 4 Cournot games. The crucial assumption for existence of a sub-game perfect equilibrium is that of finite action in stages 1 and 3 (capital decision stages). With a finite action space, an equilibrium exists by Nash's theorem.

One important characteristic of the equilibrium is that the periods are strategically independent. That is, the period-2 equilibrium play is independent of the period-1

Exhibit 5

The extensive form of period 1 of the capacity choice game



Note: The extensive form of period 2 of the capacity choice game is identical to the extensive form of the game in period 1.

equilibrium play, and vice versa. (The argument is similar to Selten's chain store paradox story, see Selten, 1978.)

Characterizing the equilibrium. We first analyze the Cournot competition stages -- stages 2 and 4. Firm i's equilibrium output is

$$Q_{it}^{*} = \frac{a - 2c_i + c_j}{3b}, \tag{5}$$

where $c_i = \bar{c}_i$ if $Q_{it}^{*} > \hat{Q}_i$ and $c_i = \underline{c}_i$ if $Q_{it}^{*} \le \hat{Q}_i$. An analogous condition holds for firm j. The equilibrium profit (induced by the pure-strategy equilibrium outputs) in stage t=2,4 for firm i is

$$\pi_{it} = \begin{cases} \frac{1}{9b}[a-2\bar{c}_i - +c_j]^2 + (\bar{c}_i - \underline{c}_i)\hat{Q}_i - \underline{F}_i & \text{if } K_{it} = K_{i0} \\ \frac{1}{9b}[a-2\underline{c}_i - +c_j]^2 - \bar{F}_i & \text{if } K_{it} = K_{i0}+x. \end{cases} \tag{6}$$

given $c_j = \underline{c}_j, \bar{c}_j$.

Due to the simultaneous capital addition/deletion decisions, there are multiple equilibria in stages 1 and 3 -- two pure-strategy equilibria and one mixed-strategy equilibrium. This creates a difficulty when examining comparative statics. We then examine what we consider to be the most interesting and plausible equilibrium -- the mixed-strategy equilibrium.

The following two inequalities are sufficient for the stage-1 and stage-3 capacity choice games to be characterized as a battle-of-the-sexes game and hence for the existence of a mixed-strategy equilibrium. First, given that firm j chooses capital K_{j0}, we assume firm i earns greater profit by choosing capital K_{i0}+x than by choosing K_{i0}. That is, the following inequality for the stage t (t=2,4) equilibrium profit holds:

$$\begin{aligned} &\pi_{it}(K_{i0}+x, K_{j0}) - \pi_{it}(K_{i0}, K_{j0}) \\ &= \left(\frac{1}{9b}[a-2\underline{c}_i + \bar{c}_j]^2 - \bar{F}_i\right) - \left(\frac{1}{9b}[a-2\bar{c}_i + \bar{c}_j]^2 + (\bar{c}_i - \underline{c}_i)\hat{Q}_i - \underline{F}_i\right) > 0. \end{aligned} \tag{7}$$

Second, given that firm j chooses capital K_{j0}+x, we assume firm i earns greater profit by choosing capital K_{i0}. That is, the following inequality for the stage t (t=2,4) equilibrium profit holds:

$$\pi_{it}(K_{i0}+x, K_{j0}+x) \; - \; \pi_{it}(K_{i0}, K_{j0}+x) \\ = (\frac{1}{9b}[a-2\underline{c}_i+\underline{c}_j]^2-\bar{F}_i) - (\frac{1}{9b}[a-2\bar{c}_i+\underline{c}_j]^2+(\bar{c}_i-\underline{c}_i)\hat{Q}_i-\underline{F}_i) < 0. \tag{8}$$

Given the demand and cost structure, Exhibit 6 lists firm i's stage-t (t=2,4} equilibrium profits as a function of the existing stage-t capacities (and hence as a function of the investments made before stage t).

We would like to note one important interpretation of the inequalities in expressions (7) and (8). The firms, prior to period 1, had the optimal capital stocks. Then, with a permanent increase in market demand at the onset of period 1, industry (i.e. joint) profit is maximized by the addition of one unit of capital. Hence, with this increase in demand and prior to any changes in capacity, the industry has undercapacity. As we demonstrate in this section, the permanent increase in market demand can generate undercapacity/overcapacity cycles.

We now analyze stages 1 and 3 assuming that the inequalities in expressions (7) and (8) hold. In stage 1, there are two pure strategies in which one firm chooses K_{i0}+x and the other chooses K_{j0}. There is also the mixed strategy equilibrium in which both firms choose K_{i0}+x with some probability. In the mixed strategy (and symmetric) equilibrium, the probability with which a firm chooses K_{i0}+x depends on his opponent's payoff structure. Firm j randomizes between choosing capital K_{j0}+x (with probability p_j) and capital K_{j0} (with probability 1-p_j) so that firm i is just indifferent to choosing capital level K_{i0} and level K_{i0}+x. The mixed strategy equilibrium requires this indifference by firm i. Otherwise, if firm i did strictly better by say K_{i0}, then it would choose K_{i0} with probability 1 and not play a mixed strategy.

To calculate the equilibrium probability, p_j^*, that firm j chooses K_{j0}+x, we set firm i's expected profit from choosing capital K_{i0}+x equal to its expected profit from choosing K_{i0}. These expected profits are determined by the equilibrium profits to the stage-2 Cournot game, and are stated in Exhibit 6. Given that firm j adds capital with probability p_j^*, firm i's expected profit from adding capital is

Exhibit 6

Firm i's state-t (t=2,4) equilibrium profit as a function of the state-t (t=2,4) capital stocks

		Firm j: $k_{jo} + x$	Firm j: k_{jo}
Firm i	$k_{io} + x$	$\frac{1}{9b}\left[a - 2\underline{c}_i + \underline{c}_j\right]^2 - \bar{F}_i$	$\frac{1}{9b}\left[a - 2\underline{c}_i + \bar{c}_j\right]^2 - \bar{F}_i$
	k_{io}	$\frac{1}{9b}\left[a - 2\bar{c}_i + \underline{c}_j\right]^2 + (\bar{c}_i - \underline{c}_i)\hat{Q}_i - \underline{F}_i$	$\frac{1}{9b}\left[a - 2\bar{c}_i + \underline{c}_j\right]^2 + (\bar{c}_i - \underline{c}_i)\hat{Q}_i - \underline{F}_i$

$$\pi_i(K_{i0}+x) = p_j^* \pi_i(K_{i0}+x, K_{j0}+x) + (1-p_j^*)\, \pi_i(K_{i0}+x, K_{j0}) \tag{9}$$

$$= p_j^* \left[\frac{1}{9b}\left[a - 2\underline{c}_i + \underline{c}_j\right]^2 - \overline{F}_i\right] + (1-p_j^*) \left[\frac{1}{9b}\left[a - 2\underline{c}_i + \underline{c}_j\right]^2 - \overline{F}_i\right].$$

and firm j's expected profit from not adding capital is

$$\pi_i(K_{i0}) = p_j^* \pi_i(K_{i0}, K_{j0}+x) + (1-p_j^*)\, \pi_i(K_{i0}, K_{j0}) \tag{10}$$

$$= p_j^* \left[\frac{1}{9b}\left[a - 2\overline{c}_i + \underline{c}_j\right]^2 + (\overline{c}_i - \underline{c}_i)\hat{Q}_i - \underline{F}_i\right]$$

$$+ (1-p_j^*) \left[\frac{1}{9b}\left[a - 2\overline{c}_i + \underline{c}_j\right]^2 + (\overline{c}_i - \underline{c}_i)\hat{Q}_i - \underline{F}_i\right].$$

Setting (9) = (10), i.e.,

$$p_j^* \pi_i(K_{i0}+x, K_{j0}+x) + (1-p_j^*)\, \pi_i(K_{i0}+x, K_{j0})$$

$$= p_j^* \pi_i(K_{i0}, K_{j0}+x) + (1-p_j^*)\, \pi_i(K_{i0}, K_{j0}),$$

and solving for p_j^*, yields

$$p_j^* = \frac{-9b(\overline{F}_i - \underline{F}_i) - 9b\,\hat{Q}_i(\overline{c}_i - \underline{c}_i) + 4a(\overline{c}_i - \underline{c}_i) + 4\overline{c}_j(\overline{c}_i - \underline{c}_i) - 4(\overline{c}_i^{\,2} - \underline{c}_i^{\,2})}{(\overline{c}_i - \underline{c}_i)(\overline{c}_j - \underline{c}_j)}. \tag{11}$$

We are interested in determining the likelihood of undercapacity/overcapacity cycles. In our model, the equilibrium strategies and payoffs in each period are identical. Therefore, suppose there is a "large" increase in demand and that each firm's capacity constraint imposes a large loss in profit. That is, each firm has a dominant strategy to choose capital K_{i0}+x and thus increase capacity in period 1. If each firm adds capital in period 1 with probability 1, then each firm optimally chooses that capital level in period 2, and there are no capacity cycles in the industry. Similarly, suppose the capacity constraint is not binding in period one and each firm has a dominant strategy to keep the present capital K_{i0} and the associated capacity. If neither firm adds capital in period 1 with probability 1, then each firm optimally chooses that capital level in period 2, and there are again no capacity cycles in the industry. We therefore do not observe capacity cycles in our model when the firms' capacity strategies are deterministic. Rather, capacity cycles occur when the firms randomize between adding capacity and not adding capacity. The firms will randomize when the capacity constraint imposes a large enough loss in profit (so that the firms do not retain their present capacity with probability 1) and not too large of a loss is profit (so that the firms do not add capacity with probability 1). In this intermediate stage of profit loss associated with the capacity constraint, the firms play a battle-of-the-sexes game in the capacity addition stage, and we observe a mixed strategy equilibrium. The probability of observing a capacity cycle is

$$\begin{aligned} \text{prob(capacity cycle)} = &\ \text{prob(both firms choose } K_{i0}\text{+x in period 1)} \\ &\times \text{prob(both firms choose } K_{i0} \text{ in period 2)} \\ &+ \text{prob(both firms choose } K_{i0} \text{ in period 1)} \\ &\times \text{prob(both firms choose } K_{i0}\text{+x in period 2).} \end{aligned} \tag{12}$$

Comparative Statics for the Mixed-Strategy Equilibrium. Now, we examine the effects of the cost structures on the probability of an overcapacity/undercapacity cycle. We first consider the effect of firm i's cost structure on the probability, in a mixed-strategy equilibrium, that firm j chooses K_{j0}+x and adds capacity. With an increase in firm i's initial capacity output, $\hat{Q}_i$, it is relatively less profitable for firm i to choose capital K_{i0}+x. That is, with the initial capital K_{i0}, capacity is not as binding and firm i has a smaller incentive to remove this capacity constraint. For firm i to remain indifferent to K_{i0} and K_{i0}+x, firm j must choose K_{j0}+x with a smaller probability. From (11), we derive

$$\frac{\partial p_j^*}{\partial \hat{Q}_i} = -\frac{9b}{4(\bar{c}_j - \underline{c}_j)} < 0. \tag{13}$$

With an increase in firm i's post-capacity marginal cost $\bar{c}_i$, it is relatively more profitable for firm i to remove the capacity constraint by choosing capital K_{i0}+x. For firm i to remain indifferent to K_{i0} and K_{i0}+x, firm j must choose K_{j0}+x with a greater probability. From (11), we derive

$$\frac{\partial p_j^*}{\partial \bar{c}_i} = \frac{-(\bar{c}_i^2 - \underline{c}_i^2) + 9b(\bar{F}_i - \underline{F}_i)}{4(\bar{c}_i - \underline{c}_i)^2(\bar{c}_j - \underline{c}_j)} > 0. \tag{14}$$

This term is positive because if $-(\bar{c}_i^2 - \underline{c}_j^2)/9b + (\bar{F}_i - \underline{F}_i) < 0$ firm i has a dominant strategy to choose K_{i0}+x and add capacity. The net fixed cost of adding capacity is small compared to the net marginal cost gains of adding capacity.

Next, with an increase in firm i's pre-capacity marginal cost, $\underline{c}_i$, it is relatively less profitable for firm i to remove the capacity constraint by choosing capital K_{i0}+x. For firm i to remain indifferent to K_{i0} and K_{i0}+x, firm j must choose K_{j0}+x with a smaller probability. From (11), we derive

$$\frac{\partial p_j^*}{\partial \underline{c}_i} = -\frac{(\bar{c}_i^2 - \underline{c}_i^2) + 9b(\bar{F}_i - \underline{F}_i)}{4(\bar{c}_i - \underline{c}_i)^2(\bar{c}_j - \underline{c}_j)} < 0. \tag{15}$$

We now consider the effect of firm j's cost structure on the probability, in a mixed-strategy equilibrium, that firm j chooses K_{j0}+x and adds capacity. With an increase in firm j's post-capacity marginal cost $\bar{c}_j$, it is relatively more profitable for firm i to remove the capacity constraint by choosing capital K_{i0}+x. For firm i to remain indifferent to K_{i0} and K_{i0}+x, firm j must choose K_{j0}+x with a greater probability. From (11), we derive

$$\frac{\partial p_j^*}{\partial \bar{c}_j} = -\frac{[(a - 2\bar{c}_i + \underline{c}_j)^2 - (a - 2\underline{c}_i + \underline{c}_j)^2] + 9b[(\bar{c}_i - \underline{c}_i)\hat{Q}_i + (\bar{F}_i - \underline{F}_i)]}{(\bar{c}_j - \underline{c}_j)^2(\bar{c}_i - \underline{c}_i)} > 0. \tag{16}$$

From the inequality in expression (8), i.e., part of our sufficient condition for a mixed-strategy equilibrium, (16) is positive.

With an increase in firm j's pre-capacity marginal cost $\underline{c}_i$, it is again relatively more profitable for firm i to remove the capacity constraint by choosing capital K_{i0}+x. For firm i to remain indifferent to K_{i0} and K_{i0}+x, firm j must choose K_{j0}+x with a greater probability. From (11), we derive

$$\frac{\partial p_j^*}{\partial \underline{c}_j} = \frac{[(a-2\bar{c}_i+\bar{c}_j)^2-(a-2\underline{c}_i+\bar{c}_j)^2]+9b[(\bar{c}_i-\underline{c}_i)\hat{Q}_i+(\bar{F}_i-\underline{F}_i)]}{(\bar{c}_j-\underline{c}_j)^2(\bar{c}_i-\underline{c}_i)} > 0. \quad (17)$$

From the inequality in expression (7), (17) is positive.

The Regions of the Mixed- and Pure-Strategy Equilibria. In Exhibit 7 we examine the effects of $(\underline{c}_j, \bar{c}_j)$ on the equilibrium strategy by firm j. In Region I_A, with c_j large and $\underline{c}_j$ small, firm j has a dominant strategy to choose K_{j0}+x and increase capacity. In Region I_A, with $\bar{c}_j$ small and $\underline{c}_j$ large, firm j has a dominant strategy to choose K_{j0} and not add capacity. In Region III_A with intermediate values of $(\underline{c}_j, \bar{c}_j)$ and a mixed-strategy equilibrium (where firm i randomizes between choosing K_{i0} and K_{i0}+x), firm j randomizes between choosing K_{j0} and K_{j0}+x. To understand the region with the mixed strategies, consider the isoprobability line labeled $p_j^* = 0.5$. This line identifies all values of $\bar{c}_j$ and $\underline{c}_j$ for which firm j chooses K_{j0} and K_{j0}+x each with probability 0.5. To derive this isoprobability line, we set the probability p_j^* as a function of $(\underline{c}_j, c_j)$ equal to 0.5. That is,

$$0.5 = p_j^*(\underline{c}_j, \bar{c}_j). \quad (18)$$

Taking the total differential of equation (18), we derive

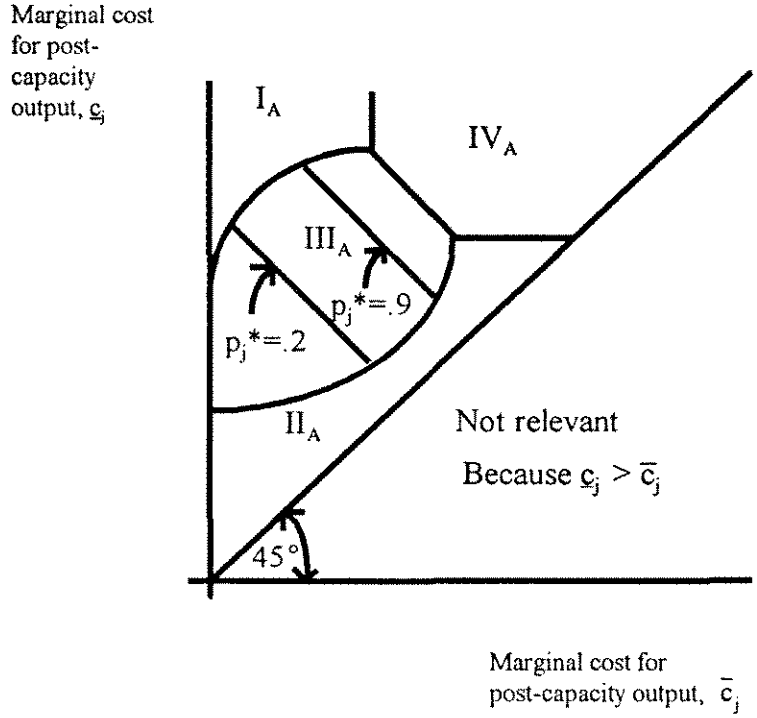
$$0 = \frac{\partial p_j^*}{\partial \underline{c}_j} d\underline{c}_j + \frac{\partial p_j^*}{\partial \bar{c}_j} d\bar{c}_j. \quad (19)$$

Then, from (19), we derive

$$\left.\frac{d\bar{c}_j}{d\underline{c}_j}\right|_{p_j^*=0.5} = -\frac{\partial p_j^*/\partial \underline{c}_j}{\partial p_j^*/\partial \bar{c}_j} < 0. \quad (20)$$

Exhibit 7

The effect of $\bar{c}_j$ and $\underline{c}_j$ on firm j's capacity decisions



Notes:

Region I_A:	Firm j has a dominant strategy to choose K_{j0}+x.
Region II_A:	Firm j has a dominant strategy to choose K_{j0}.
Region III_A:	There is a mixed-strategy equilibrium. Two isoprobability lines are labeled.
Region IV_A:	Firm j exits.

Similarly, the slope of the each of the isoprobability lines is negative. Also, from expressions (7) and (8), p_j^* is increasing in $(\underline{c}_j, \bar{c}_j)$. In Region IV_A, costs are sufficiently large so that firm j exits the industry.

In Exhibit 8 we examine the effects of $(\hat{Q}_i, \bar{c}_i)$ on the equilibrium strategy by firm j. For this diagram, we consider the case for which (7) and (8) are satisfied, and hence firm j does not have a dominant strategy. In Region I_B, with $\hat{Q}_i$ small and $\bar{c}_i$ large, firm i has a dominant strategy to choose K_{i0}+x and add capacity. Firm j optimally responds to i's choice of K_{i0}+x by choosing K_{j0}. In Region II_B, with $\hat{Q}_i$ large and $\bar{c}_i$ small, firm i has a dominant strategy to choose K_{i0} and not add capacity. Firm j optimally responds by choosing K_{j0}+x and adding capacity. In Region III_B, with intermediate values of $\bar{c}_j/\hat{Q}_i$, there is a mixed-strategy equilibrium. The slope of an isoprobability line is

$$\left.\frac{d\bar{c}_j}{d\hat{Q}_i}\right|_{p_j^* \text{ constant}} = - \frac{\partial p_j^*/\partial \hat{Q}_i}{\partial p_j^*/\partial \bar{c}_j} > 0. \tag{21}$$

Moreover, p_j^* is increasing in $\bar{c}_i$ and decreasing in $\hat{Q}_i$.

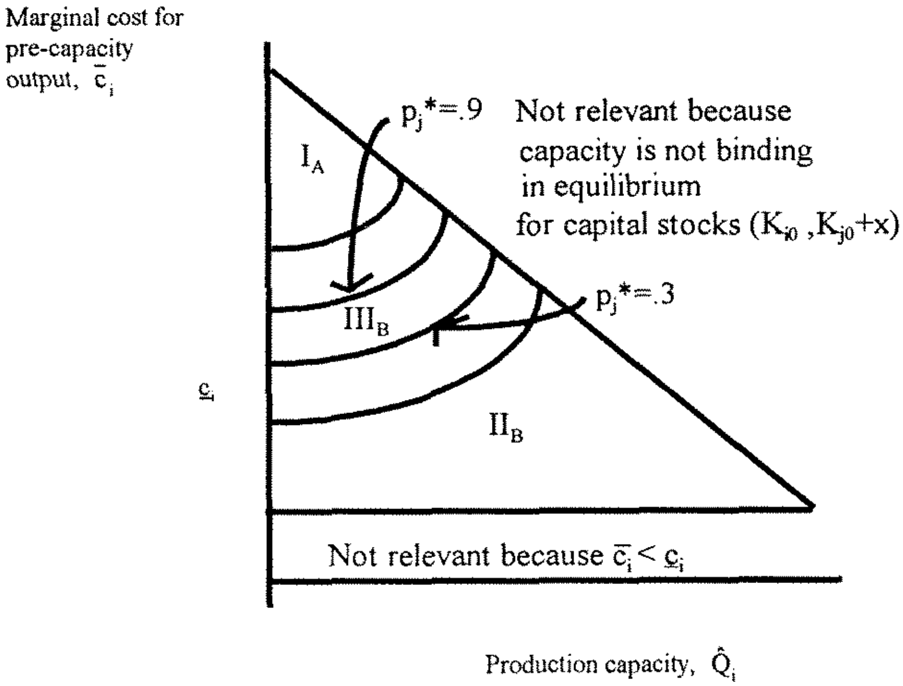
In Exhibit 9 we examine the effects of $(\hat{Q}_i, \underline{c}_i)$ on the equilibrium strategy of firm j. For this diagram, we again consider the case for which (7) and (8) hold, and hence firm j does not have a dominant strategy. In Region I_C, with both $\hat{Q}_i$ and $\underline{c}_i$ large, firm i has a dominant strategy to choose K_{i0}+x and add capacity. Firm j optimally responds to i's choice of K_{i0}+x by choosing K_{j0}. In Region II_C, with both $\hat{Q}_i$ and $\bar{c}_i$ small, firm i has a dominant strategy to choose K_{i0} and not add capacity. Firm j optimally responds by choosing K_{j0}+x and adding capacity. In Region III_C, with intermediate values of $\bar{c}_j + \hat{Q}_i$, there is a mixed-strategy equilibrium. The slope of an isoprobability line is

$$\left.\frac{d\underline{c}_j}{d\hat{Q}_i}\right|_{p_j^* \text{ constant}} = - \frac{\partial p_j^*/\partial \hat{Q}_i}{\partial p_j^*/\partial \underline{c}_j} < 0. \tag{22}$$

Moreover, p_j^* is decreasing in $(\bar{c}_i, \hat{Q}_i)$.

Exhibit 8

The effect of $(\hat{Q}_i, \bar{c}_i)$ on firm j's strategy.



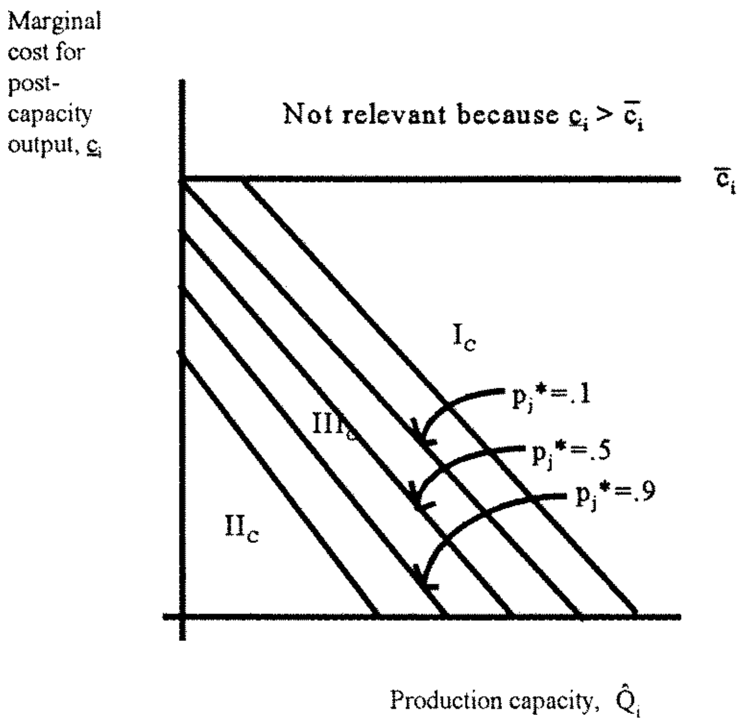
Notes:

This figure considers the case in which firm j does not have a dominant strategy.

Region I_B:	Firm i has a dominant strategy to choose K_{i0}+x. Firm j best responds by choosing K_{j0}.
Region II_B:	Firm i has a dominant strategy to choose K_{i0}. Firm j best responds by choosing K_{j0}+x.
Region III_B:	Mixed-strategy equilibrium region. Representative isoprobability lines are labeled.

Exhibit 9

The effect of $(\hat{Q}_i, \underline{c}_i)$ on firm j's strategy



Notes:

This figure describes the case for which firm j does not have a dominant strategy.

Region I_C:	Firm i has a dominant strategy to choose K_{i0}. Firm j best responds by choosing K_{j0}+x.
Region II_C:	Firm i has a dominant strategy to choose K_{i0}+x. Firm j best responds by choosing K_{j0}.
Region III_C:	Mixed-strategy equilibrium region. Representative isoprobability lines are labeled.

To further understand the different regions of the equilibrium in Exhibit 9, fix $\hat{Q}_i$ and increase $\underline{c}_i$ so that the equilibrium moves from region II_C to region III_C. With this movement, there is a discontintuity in firm j's equilibrium strategy. In region II_C, firm i has a dominant strategy to choose capital K_{i0} +x and add capacity. Firm j best-responds by choosing capital K_{j0} and not adding capacity. With an increase in $\underline{c}_i$ and the movement to slightly above the boundary of regions II_C and III_C. Now, firm i no longer has a dominant strategy to add capacity. However, $\underline{c}_i$ is small enough so that firm i has a strong incentive to choose capital K_{i0}+x and add capacity. For firm i to remain indifferent between between adding and not adding capacity, firm j must add capacity with a large probability. Thus, with the movement from region II_C to region III_C, firm j's equilibrium strategy jumps from adding capacity with probability zero to adding capacity with a very large probability. There is an analogous discontintuity in firm i's equilibrium strategy with the movement from region III_C to region I_C.

4.3. Model Assessment

Our theoretical model admits a mixed strategy equilibrium, with both firms randomizing their capacity addition and deletion decisions, yielding a positive probability of capacity cycles. Our comparative statistics show how the likelihood of such cycles in theory can be affected by costs and capacity utilization.

How do these results compare to the dynamics of the markets in general or to those studied in Dearden, Lilien and Yoon (1996), TiO_2 in particular? Consider Exhibit 10, which suggests several behaviors that are consistent with our model.

<u>Observation 1</u>: We see that individual firms' *simultaneous* capacity additions and maintenance (e.g., 1973-74 or 1982-83) are all followed by their *simultaneous* capacity deletions or maintenance (e.g., 1976-76 or 1984-85).

<u>Observation 2</u>: The cycles (i.e., additions and deletions) of total industry capacity have occurred in parallel with the cycles (i.e., Highs and Lows) of *capacity utilization* with a typical lag of 1-2 years.

<u>Observation 3</u>: Low and medium *cost* firms (e.g., A or B) have typically responded to industry fluctuations by their capacity cycles of additions and maintenance, while *high* cost firms (e.g., C and D)

have responded to industry fluctuations by their capacity cycles of additions, *deletions*, and maintenance.

To generate an additional observation we focus on firms A and C, the "high" market share firms who mainly dictate the evolution of the market. If we assume that A and C are playing fixed strategies, then we would expect to see certain deterministic patterns of simultaneous moves dominate. (Both add, both delete, A adds, C doesn't, etc.) If we code the data as follows:

1. A+2	=	A adds, C deletes	[3]
2. A+1	=	{A adds, C no change; or A no change ,C deletes	[6]
3. A	=	{A adds, C deletes; or A no change, C no change; or A deletes, C adds	[6]
4. A−1	=	{A no change, C adds; or A deletes, C no change	[1]
5. A−2	=	A deletes, C adds	[0]

we have five possible "deterministic strategy" patterns relating A's and C's capacity strategies. For a deterministic strategy to dominate, we should expect one of these patterns to prevail.

The numbers in [ ] show the frequency with which the noted pattern occurs in Exhibit 10. Not surprisingly, the A - 1 and A - 2 patterns were virtually nonexistent (since A is the low cost firm). However, the high incidence of all three of the other patterns (3, 6 and 6) suggest capacity decisions that look like mixed strategies, providing qualitative support for our model.

5. Discussion and Conclusions

The dynamics of markets appear to lead to cycles of overcapacity and undercapacity. In this paper we have explored some of the forces that lead to those cycles.

Exhibit 10
Capacity Additions and Deletions* in the TiO_2 Industry

Year	Individual firm's capacity						aggregate industry capacity	capacity utilization**	
	A	B	C	D	E	F		1-yr lag	2-yr lag
1970	+	+	o	+	+	+	+	H	M
71	o	o	o	-	o	+	+	M	H
72	+	o	-	+	o	+	-	M	M
73	+	o	-	+	o	+	+	H	M
74	+	+	+	o	+	o	+	H	H
75	o	o	o	o	+	o	+	M	H
76	+	o	-	o	o	o	+	L	M
77	o	o	-	o	+	o	-	L	L
78	o	o	-	o	o	o	-	L	L
79	+	o	o	-	o	o	-	M	L
80	+	o	o	o	o	o	+	M	M
81	-	o	-	-	+	+	+	M	M
82	o	+	o	+	o	+	+	M	M
83	o	+	+	+	+	+	+	L	M
84	o	o	-	o	+	o	-	M	L
85	o	o	o	o	o	o	+	H	M
Individual Firm's Position									
(a)Production cost***									
	L	L	H	H	M	M			
(b)Market share****									
	H	L	H	M	M	L			

* +: addition, -: deletion, and o: no change.
** H: 90% or higher, M: 80-90%, and L: 80% or lower.
*** H: relatively high, M: medium, and L: relatively low.
**** H: relatively high, M: medium, and L: relatively low.

These cycles are inefficient, whatever their causes, disrupting buyers, sellers and often leaving ultimate consumers with either higher prices or delayed access to products desired. What can sellers do to address these issues? Bower (1986, p. 221) suggests: "...it would be extremely helpful...if industry associations were asked to produce long term forecasts of supply/demand balance." What Bower suggests is that reducing uncertainty about the nature of demand would help coordinate capacity planning efforts. This will surely help, but our game-theoretic results indicate that unless individual company plans are coordinated, we are unlikely to see a cure.

Are there alternatives? In Japan, MITI helps coordinate the strategic plans of competitive companies. Firms give up some independence (legally in Japan, at least) in exchange for the benefits of coordination or consensus building. The stabilizing effects of such cartel-like coordination procedures reduce cyclical behavior but may do so at the cost of keeping inefficient producers in the market (Shaw and Shaw, 1983).

In the absence of the possibility of such coordination in many of Western industrialized nations, firms need to adopt other strategies. This paper serves as further support for the risks of operating in such markets: overcapacity/ undercapacity cycles are almost destined to occur. Flexible manufacturing systems, sharing or reallocating production capacity with other, countercyclical (or uncorrelated, at least) products can help, though (Breshnahan and Ramey, 1993). In addition, firms may need to budget for higher expected returns to deal with the risks inherent in operating in such industries.

Firms can take advantage of these situations. Although our models have not addressed the issue, there is clearly value in information about demand and likely competitive actions. Better industry and competitive intelligence is likely to pay large dividends for firms operating in such markets, signing longer terms supply contracts with customers during the onset of undercapacity, for example. We also speculate that there may be advantages to various forms of bluffing (announcing expansion/deletion plans for strategic rather than operational reasons).

There may well be regulatory implications here. If there were agreement on the likely level of demand, one could envision a situation where the government entertained bids for the rights to add new capacity and limited the amount of capacity to some multiple of the industry's estimate of the increase in demand. There are clearly many problems with the development of such a system, as the US government's recent experience with cellular phone franchises and bandwidth auctions has shown, but the idea may have some merit.

Our model and analysis and the above speculations have been exploratory. We have investigated only a simple model here (see Dearden, Lilien, and Yoon, 1996, for others) and one can envision many other causes of the capacity cycle phenomenon. We do not believe that this phenomenon has a single cause or set of causes; rather we believe that it would be valuable, in future research, to see how general the phenomenon is and to generate and develop a taxonomy of causes and possible cures. We hope that we have shed some light on some of the possible causes and that further work helps deepen that understanding.

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