Why Bass Model Estimates May Be Biased (and What It Means)

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1. Introduction

Diffusion studies in which Bass models are estimated using aggregate data typically feature two intriguing results. First, the estimated population size is close to the cumulative number of adopters observed in the last time period for which data are available. In some cases, there is evidence that the estimate is much smaller than the true population size. Second, the estimated contagion effect is very strong, which is at variance with research findings in consumer behavior and other social sciences. We suggest these two empirical regularities may be related. More specifically, we suggest that underestimating the population size is similar to censoring away consumers or firms that have not adopted yet, which in turn results in an upward bias in the contagion parameter.

We first review some of the evidence for the two empirical regularities. Next, we present the intuition showing how they can be related to one another via a right censoring mechanism. Third, we identify several implications these biases have for marketing strategy, forecasting, and product life cycle theory. We conclude by outlining how to assess the severity of these biases using both synthetic and empirical data.

2. Two intriguing empirical regularities

Innovation diffusion models in marketing and sociology are based on the behavioral assumption that new product acceptance is an imitation or social contagion process. Although

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diffusion modelling has become a vibrant research tradition, most empirical work still show the structure of the basic epidemic model introduced by Coleman (1964) and Bass (1969). Empirical diffusion studies in marketing have typically followed the Bass approach in which aggregate sales or penetration data are used to calibrate a three parameter model of the form:

$$dX(t) / dt = [\, p + qx(t) \,] \, [\, m - X(t) \,],$$

which, if m is known, can be rewritten as,

$$dx(t) / dt = [\, p + qx(t) \,] \, [\, 1 - x(t) \,],$$

where $X(t)$ is the number and $x(t)$ is the proportion of people having adopted by time t, and m is the size of the population $(x(t) \equiv X(t)/m)$. The factor $[\, p + qx(t) \,]$ represents the hazard rate, which is the limiting probability that an individual who has not adopted yet, does so at time t. The coefficient q reflects the extent to which this probability increases with the proportion of the population that has already adopted. The rationale behind the model is that the higher the number of previous adopters, the stronger the social contagion within the population, and thus the higher the probability of adoption. The coefficient p reflects the fixed component, and captures individuals' intrinsic tendency to adopt as well as the effect of constant external influences such as advertising. In the marketing literature, p is commonly referred to as the coefficient of external influence, and q as the coefficient of internal influence.

Marketing modellers have traditionally estimated the three-parameter Bass model using a single time series of aggregate-level sales data. Originally, researchers used OLS, but more recently nonlinear least squares (NLS) has become the estimation procedure of choice. This three-parameter, macro-level modelling approach has gained wide-spread acceptance (for reviews, see Mahajan, Muller and Bass 1993; Parker 1994).

A review of the many studies reported since the original Bass paper suggests two intriguing regularities: the population size m tends to be systematically underestimated, and the ratio of the estimated influence parameters q/p tends to be very high.

2.1. Is population size underestimated?

Empirical diffusion studies in marketing have typically reported estimates for the size of the population of potential adopters (referred to in hazard rate analysis as the "population at risk") that are very close to the number of people having actually adopted by the last observation period, rather than the true population size. Table 1 presents actual data and NLS estimates for a few product categories that have been extensively studied and for which the true size of the population at risk was known exogenously.

Past studies tend to interpret m as the ultimate penetration level that the innovation can be expected to achieve. Following this interpretation, the estimate for m could be smaller than the true population from which the data are collected. However, such an interpretation is not statistically sound. Since the Bass model is a two-state hazard rate model with one state being absorbing and no population heterogeneity, it bears the logical requirement that the entire population from which the data are collected ultimately makes the transition. Thus, given that the model specification is true, the expected value of the estimated m must equal the entire population. Table 1 suggests that the NLS estimates do not meet this requirement. There are strong indications of a systematic downwards bias in the estimation procedure.

Table 1. The NLS estimate for population size is seriously biased downwards

	cumulative adopters in last observation period	estimated population size	true population size
Tetracycline	105	105.5	125
Ultrasound scanners	168	167.4	209
Mammography scanners	119	111.4	209
Foreign language	36	37.6	100
Accelerated program	64	64.4	100
Room air conditioners	144	187	533
Clothes dryers	123	165	533
Color televisions	323	397	654

Source: The tetracycline data and results are taken from Burt (1987). All other information is taken from Mahajan, Mason, and Srinivasan (1986), except for the true population size for the last three products which is taken from Schmittlein and Mahajan (1982).

2.2. Is social contagion overestimated?

Empirical diffusion studies in marketing have typically reported strong social contagion effects. A meta-analysis of 15 different studies estimating 213 Bass models with aggregate data has reported an average value of 0.38 for the coefficient of internal influence q, and only 0.03 for the coefficient of external influence p (Sultan, Farley, and Lehman 1990). This gives an average p/q ratio of 1 over 12. However, to put both coefficients on the same scale, we need to compare p with $qx(t)$. At the midpoint of the diffusion cycle, when $x(t) = 0.5$, $qx(t)$ is six times greater than p, suggesting that social contagion is about six times as powerful as people's intrinsic tendencies and the invariant marketing influences they are subject to. Such a strong influence of the social environment on individual behavior is at variance with the existing knowledge about consumer behavior. For instance, a review of 44 applications of Fishbein and Ajzen's model of reasoned action (Ajzen 1985; Ryan and Bonfield 1975) indicates that normative social pressure does not have a larger effect on people's choice behavior than attitude, i.e. individual assessment of the choice alternative. In a review of 19 tests of Ajzen's theory of planned behavior, attitude had a significant effect on behavioral intention in 18 cases, whereas normative pressure had a significant effect in only 8 cases. Attitude had a smaller regression coefficient in only a single instance (Ajzen 1991). Overall, behavioral research in social psychology and marketing indicates that social influence is not as large a factor in consumer behavior as Bass estimates suggest.

How, then, can such strong contagion estimates be observed over and over again in marketing diffusion studies? We posit that research calibrating three-parameter Bass-type models is subject to right censoring which inflates the estimate of the coefficient of internal influence q.

2.3. Are both phenomena related?

Based on our recent experiences with divergent results obtained through estimating the three-parameter Bass model on aggregated data versus those obtained through direct estimation of the hazard function using disaggregate data (Van den Bulte and Lilien 1994), we posit the following conjectures:

1. the NLS estimate of m is biased downwards toward $X(t^*)$, where t^* indexes the last observation period;
2. *the downward bias in m generates an upward bias in q.*

We are not aware of any strong a priori theoretical-statistical reason to expect the first conjecture to be true, except for the fact that all the information available in the data set is bounded from above by $X(t^*) < m$. In any case, Table 1 suggests that downward bias in m is common.

The second conjecture rests on firmer ground. Limiting m to $X(t^*)$ amounts to excluding from the estimation procedure the information that [m - X(t*)] individuals were censored, i.e. had not adopted by the end of the observation period. The exclusion of censored cases from the estimation sample of a hazard rate model can create large biases (e.g., Tuma and Hannan 1979). We have started some analytics as well as some simulations to assess the nature of this bias for the Bass hazard rate. Both approaches point to the same conclusion: imposing a population size lower than the true one generates an artificial upward shift in the time-path of the estimated hazard function. Also, evidence that the estimates of m and q are negatively correlated has already appeared in the marketing literature, but has not received much attention (e.g., Jain and Rao 1990, Tables 2 and 3; Jones and Ritz 1991; see also Parker 1993, p. 91). Relatedly, Balasubramanian and Ghosh (1992) reported evidence that t^* and the estimated q are negatively correlated in the Non-Uniform Influence model presented by Easingwood, Mahajan and Muller (1983). This model has hazard rate $p + qx[(t)]^{\delta}$ and simplifies to the Bass model when $\delta = 1$.

If our conjectures are indeed supported and if the biases in m and q are substantial, there are significant managerial and theoretical implications.

3. Why should we care?

To theoretically oriented scholars, estimation bias is a major concern. Biased estimates imply one's conclusions are not statistically valid, and thus do not contribute to theory development. However, there are also more pragmatic reasons why one should care about the existence of biases in diffusion parameters.

- Product and market development. Managers interpreting the estimated m as an indication of the ultimate penetration a new product or technology can be expected to achieve, may seriously underestimate that market potential. Especially worrisome is the pattern exhibited in Table 1 where "the estimated market potential" is close to the market size achieved at the time of the study. As a consequence, insufficient resources may be invested in product and market development, so that the belief that the market is close to saturation becomes a self-fulfilling prophecy.
- Marketing mix strategies. Most normative conclusions arrived at by analytical modelling of diffusion patterns, hinge on the relative size of p, q, cost dynamics, and

marketing mix effects. These factors determine the optimal decisions about skimming vs. penetration strategies, and the time path for price and advertising for a new product. Because the information for making such decisions is often inferred from previous research for similar products, managers would be well advised not to follow normative modelling conclusions unless they can be confident that the parameter values published previously are at least approximately correct. For instance, the market introduction strategy that Philips took for launching their digital compact cassette (DCC) hinged on trickle-down effects from music buffs to the general audience. Sony, on the other hand, went after the mass market faster when launching its mini-CD (Balasubramanian et al. 1994; Sherwood 1994). Which strategy is most efficient will partly hinge on how strongly adoption is driven by social contagion.

- Forecasting. Biased parameter estimates may also explain why Bass models tend to fit extremely well on their estimation sample (R^2s above 95% are quite frequent), but to exhibit poor forecasting performance. This, in turn, suggests that forecasting applications may benefit from using exogenous information on the population size, or from using a generalized closed form solution, explicitly featuring a ceiling lower than the size of the entire estimation sample (see Mahajan and Schoeman 1977, p. 15).
- Macro vs. micro modelling. As more micro-level adoption data become available, marketing researchers will increasingly use micro-level modelling techniques (e.g., Weerahandi and Dalal 1992). We conjecture that this new research approach will typically produce lower contagion estimates because they appropriately take care of right censored data. One recent application, for instance, did not report a significant contagion effect when using a micro-level estimation procedure (Van den Bulte and Lilien 1994), whereas applying the macro-level NLS estimation procedure did produce a significant imitation coefficient (Burt 1987). A careful study of upward bias of q produced by traditional estimation techniques would help understanding why such divergences between micro and macro level modelling occur.
- PLC theory. Consumer-based explanations of the product life cycle hinge on the existence of word-of-mouth and other social contagion effects (Mahajan 1994). If such effects are lower than currently believed, then technology-based and competition-based explanations should be given much more credence. This, in turn, would imply that firms, acting in isolation or in concert, have much greater degrees of freedom in influencing the PLC than is typically asserted.

4. Further analysis to follow

We presented reasons to expect that the three-parameter Bass model estimates of market size and contagion effect are biased. The bias we discussed is an estimation bias, not a model misspecification bias. Therefore, we expect it to occur even when all assumptions undergirding the Bass model are met, such as population homogeneity, perfect random mixing, constant market size, no impact of time-varying marketing variables, etc. (cf. Mahajan, Muller and Bass 1993, pp. 360-369).

Whether and to what extent the NLS estimates are biased remain analytical and empirical questions. We plan to address these through analysis of both empirical and simulated data. Products for empirical analysis will include office equipment in multiple industries, medical equipment, the tetracycline drug, and contraceptives. In the empirical analysis, we will only use data for which the true size of the population at risk is known and fixed over the entire time period. Even with this precaution, one cannot expect that the empirical data truly behave according to the Bass specification. As a consequence, one is not able to discriminate with certainty the extent to which any bias found in the stimation of m is due to misspecification rather than estimation. Also, no conclusion about q is possible since we don't know its true population value. Therefore, we will also assess the estimation bias through analysis of synthetic data generated from the Bass model. This will provide unambiguous information about estimation bias for both m and q.

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