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The Implications of Diffusion Models for Accelerating the Diffusion of Innovation

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ABSTRACT

This paper explores the implications of a simple, yet robust model of innovation diffusion for developing insight into the problem of controlling the rate of new product diffusion. Some basic, theoretical results are developed using a simple model. Those results are shown to relate to optimal policies developed from a more complex model of innovation diffusion, developed for the Department of Energy's photovoltaic program.

Introduction

Much work has been done recently (see [8, 14] for reviews) on the development of mathematical models to describe the diffusion of new products and technologies. Many of these models are quite good at describing the time path of the innovations, after the fact. But, with few exceptions [6, 7, 12], these models have not focused on ways of accelerating or controlling the diffusion rate over time or over regions.

This paper has two objectives. The first is to demonstrate that a widely referred to and used model of innovation diffusion has some interesting implications (some questionable) for accelerating the rate of new product diffusion. The second is to show that those insights are consistent with the type of results we obtain for a more detailed model, currently being developed to assess government development program options for solar generated electricity (photovoltaics).

The Timing of Demonstration Programs

Bass [2] introduced a flexible diffusion model that has useful behavioral implications. We choose this model for analysis as it has been applied by Bass and others [5] to a number of product situations and has been shown to work quite well.

We introduce the following notation:

$S(t)$ = number of firms having adopted an innovation by time t $(S(0) = 0)$.

S^* = total number of firms considered eligible to adopt the innovation.

p = coefficient of innovation; this equals the rate of product adoption when there have been no previous purchases.

q = coefficient of imitation; the effect of previous purchases on the rate of adoption.

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In essence, then, Bass's model is as follows:

$$\frac{dS(\mathrm{T})}{dt} = \left[p + q\frac{S(t)}{S^*}\right][S^* - S(t)]. \tag{1}$$

As formulated here, this model has no controllable variables. Let us consider the problem that the government (or a private sector marketer) faces when deciding how to accelerate or control this process. We take the point of view of the government, where the government can develop what we shall call "demonstration programs." Let

$A(t)$ = number of government-sponsored demonstration programs installed by time t. In private sector terms these marketer-placed units are called "reference installations"

Note that the class of demonstration programs that is most appropriate for analysis here are of the "cooperative" or "government-support" types. Here an agency of the government requests a builder/developer and a buyer to submit a proposal for a project. The government shares the cost and, from time to time, will inspect and monitor the performance of the system. Most such projects show little external sign of being government sponsored. By design they are supposed to be similar to private sector-purchased systems.

Thus it is not unreasonable to assume that imitators in eq. (1) are equally impressed by any successful project, whether it be government sponsored or privately owned. We also assume that the demonstration programs affect neither the coefficient of innovation p nor the coefficient of imitation q.[1] Following these assumptions, we get

$$\frac{dS(t)}{dt} = \left[p + q^*T(t)\right]\left[S^* - S(t)\right], \tag{2}$$

where

$$T(t) = S(t) + A(t) \quad \text{and} \quad q^* = q/S^*.$$

Suppose that $A(\infty) = C$ (the government will ultimately set up a total of C demonstration programs). Our first question is how, given this model, these installations should be timed to hasten the diffusion of the new product. This is analyzed as follows.

The form of eq. (2) allows separation of $dS(t)/dt$ into two components.

$$\frac{dS(t)}{dt} = \frac{dS_1(t)}{dt} + \frac{dS_2(t)}{dt},$$

where

$$\frac{dS_1(t)}{dt} = p\left[S^* - S(t)\right] + \frac{q}{S^*}S(t)\left[S^* - S(t)\right]$$

and

$$\frac{dS_2(t)}{dt} = \frac{q}{S^*}A(t)\left[S^* - S(T)\right].$$

Since, at any time t, $dS_2(t)/dt$ is greater when $A(t)$ is greater, $dS_2(t)/dt$ (and hence

[1] See Kalish and Lilien [9] for some suggestions on how to relax these assumptions.

$dS(\bar{t})/dt$) will be maximal when all demonstration program resources are allocated as early as possible.

According to this model, then, it cannot pay to delay allocation of demonstration resources. In considering allocation of effort across sectors of the economy or across regions, with this model we need only be concerned with the *initial* level of support (i.e., at $t = 0$), since any delay can be improved, as above, by acceleration in time.

Allocation over Sectors

Let us now assume a set of diffusion curves, each one relevant for a sector (agriculture, commercial, industrial consumer, etc.) or a geographically separated region (northeast, etc.). We assume here that q, the coefficient of imitation, is a function of the level of demonstration-program support A, so that

$$q = f(A)$$

or

$$\frac{dS_i(t)}{dt} = \left[S_i^* - S_i(t)\right]\left[p_i + f_i(A_i(t))\, T(t)\right],$$

where i refers to a sector or region. As we are only concerned with $t = 0$, this reduces to

$$\frac{dS_i(0)}{dt} = S_i^*\left[p_i + A_i f_i(A_i)\right], \qquad (3)$$

where

$$\acute{A}_i = A_i(0).$$

If we let

$$y_i = \frac{dS_i(0)}{dt} = \text{adoption rate at time 0}$$

and

$$d(A_i) = A_i f(A_i),$$

we get

$$y_i = p_i S_i^* + d(A_i) S_i^*. \qquad (4)$$

If we wish to maximize the sales rate, then we must maximize $\sum y_i$ subject to a government budget constraint $\sum A_i \leqslant K$.

As a first case, consider two equal sectors.

Here we have

$$p_1 = p_2 = p, \qquad d_1 = d_2 = d, \qquad S_1^* = S_2^* = S.$$

The problem can be formulated as

find	A_1 and A_2
to maximize	$(y_1 + y_2)/S$
subject to	$A_1 + A_2 \leqslant K.$

This is the same as

find A_1 and A_2

to maximize $2p + d(A_1) + d(A_2)$

subject to $A_1 + A_2 \leqslant K$.

Theorem 1. If d is concave, the optimal policy is to allocate equally.[2]

Our conclusion is that if the imitation parameter is a concave function of the number of demonstration installations, the optimal allocation is an equal or spread-out development policy.

Theorem 2. If d is convex, develop only one sector.

Similarly, for N sectors, the optimum policy is to allocate all resources to one sector.

Perhaps the most reasonable assumption about the shape of d is that it has a convex, then a concave region. Thus the first few demonstration projects show increasing returns, then the effects gradually diminish (an S-shaped response).

Here the analysis is more complex; let us deal with the case of N sectors directly. Construct a concave envelope over d. We refer here to Figure 1, where A^* is the tangent point. Consider three cases.

Case 1. If $A^* > K/N$, then if we use the concave envelope to approximate d, the solution falls in the sector in which the concave envelope and the function d correspond. This corresponds to Theorem 1 and $A_i = K/N$ for all i.

Case 2. If $A^* > K$, then the optimal solution is to allocate all resources to one region. This follows since the return per unit of A increases, in each area, up to point A^*, by definition of the tangent point.

Case 3. If $K > A^* > K/N$, the solution is nontrivial and must be calculated for the particular functional form of d. However, we can get an approximate solution by treating all d as if they were represented by their concave envelopes. In this case (as long as resources are allocated to markets one at a time), the solution, with the d then concave, will be suboptimal by at most $U = \max_A \left[A\big(f(A^*)/A\big) - f(A)\right]$. Thus we have a near-optimal solution here with an upper bound U on its deviation from optimal.

We now investigate the case of N regions or sectors with different response parameters.

Theorem 3. If all d_i are concave, an allocation plan according to incremental innovator returns will yield an optimal allocation policy.

Theorem 4. If all d_i are convex, an allocation plan in one market is optimal.

Suppose, once again, that all d_i are S-shaped. A near-optimal policy can be constructed using the same procedure as outlined in case 3. A solution obtained in this way (due to the linear, concave envelope) will lead to a policy of allocating resources to sectors one at a time until the strictly concave region is reached.

It is possible that the solution obtained in this way will lie on the concave envelope for at most one sector. In this case, the solution will be suboptimal by no more than

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[2] All proofs of theorems are in the appendix.

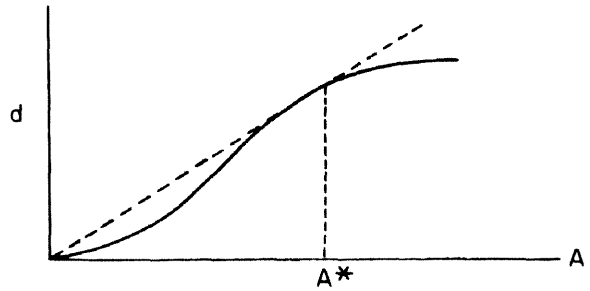


Fig. 1. An S-shaped response and a concave envelope.

$$U = \max_{i,A_i}\left[Af_i(A_i^*)/A_i^* - f_i(A_i)\right].$$

Once again, the suboptimality of this solution is bounded.

The results here are quite interesting. As we shall see in the next section, they provide initial guidance for program planning in more realistic situations. In particular, we have shown that, following from a Bass-type model,

1. Delaying marketing development resources is unlikely to be effective. This assumes no customer feedback will be used to modify the product.
2. Concave imitation response implies a spread-out strategy. Suppose the effect of each installation on market acceptance were less than the effect of the previous one. Market resources should then be spread as widely as possible.
3. Convex imitator response implies a concentrated strategy. Suppose the second application had more effect on potential buyers than the first, the third more than the second, and so on. In this case, all resources should be concentrated in a single sector.
4. S-shaped response implies concentrating in one area at a time, then spreading out. S-shaped response combines the early effects of convex response with the later effects of concave response. This case suggests building up a sector to a point where it is self-sustaining and then going up.

The economics of purchase and varying market potentials have not been considered here. However, the results, although not meant to be definitive, do suggest conditions under which certain policies—over regions, market segments, and time—are preferable to others.

There are several key assumptions in this analysis. First, as discussed earlier, we assume that demonstration program resources have the same effect as private purchases. Second, we assume no "neighborhood" effects across sectors. This is equivalent to asking a farmer, "What would be the influence of adoption of photovoltaics in a pharmaceutical plant on your likelihood of adoption?" Field survey work [10] suggests this effect is small enough to be ignored in the analysis we are performing here. This may not be the case for other innovations and is discussed by Mahajan and Peterson [14].

In addition, we have considered neither the economics of purchase nor the effects of varying market potential here. However, our intent is to develop insight into conditions under which certain policies—over markets and time—might be preferable to others. This will help us in more realistic cases in both calculating optimal policies and in explaining their structure.

Implications for Policy Analysis: The Photovoltaics Case

The results derived in the last section are useful in two ways: First, note that we shall develop a more realistic policy evaluation model here. That model searched for "good" government policies and, in optimization mode, requires reasonable starting points for evaluation. The models in the previous section give good starting points that lead to quick computational convergence.

Second, the previous analysis suggests why the results of even these complex models should follow specific forms. As most solar energy penetration models (see Warren [18] for a review) are little more than a series of simple diffusion models linked together, the previous analysis suggests why certain policies will emerge from these models as preferable to others.

THE MODEL

The objective of the model, called PVO, is to provide a tool for evaluating the impact of government programs on the diffusion rate of photovoltaics. The model considers the following controlling influences:

1. cost per unit of energy produced,
2. the perception of risk in adopting the innovation,
3. external factors, such as government policy, and
4. noneconomic factors such as aesthetics and "energy saving."

The model predicts total cumulative demand as a function of controllable parameters. For example, it shows the effect of a marketing policy that reduces perceptions of risk. It determines how the government can best allocate resources to stimulate demand. There are two related assumptions underlying the PVO model:

1. Cost (price) declines in some way as production increases, increasing potential demand.
2. The likelihood of adoption in a sector is increased as the number of successful installations increases. In this model, demonstration projects are assumed to affect potential adopters in the same environment or "sector," suggesting that we split the market into different sectors, each with separate response.

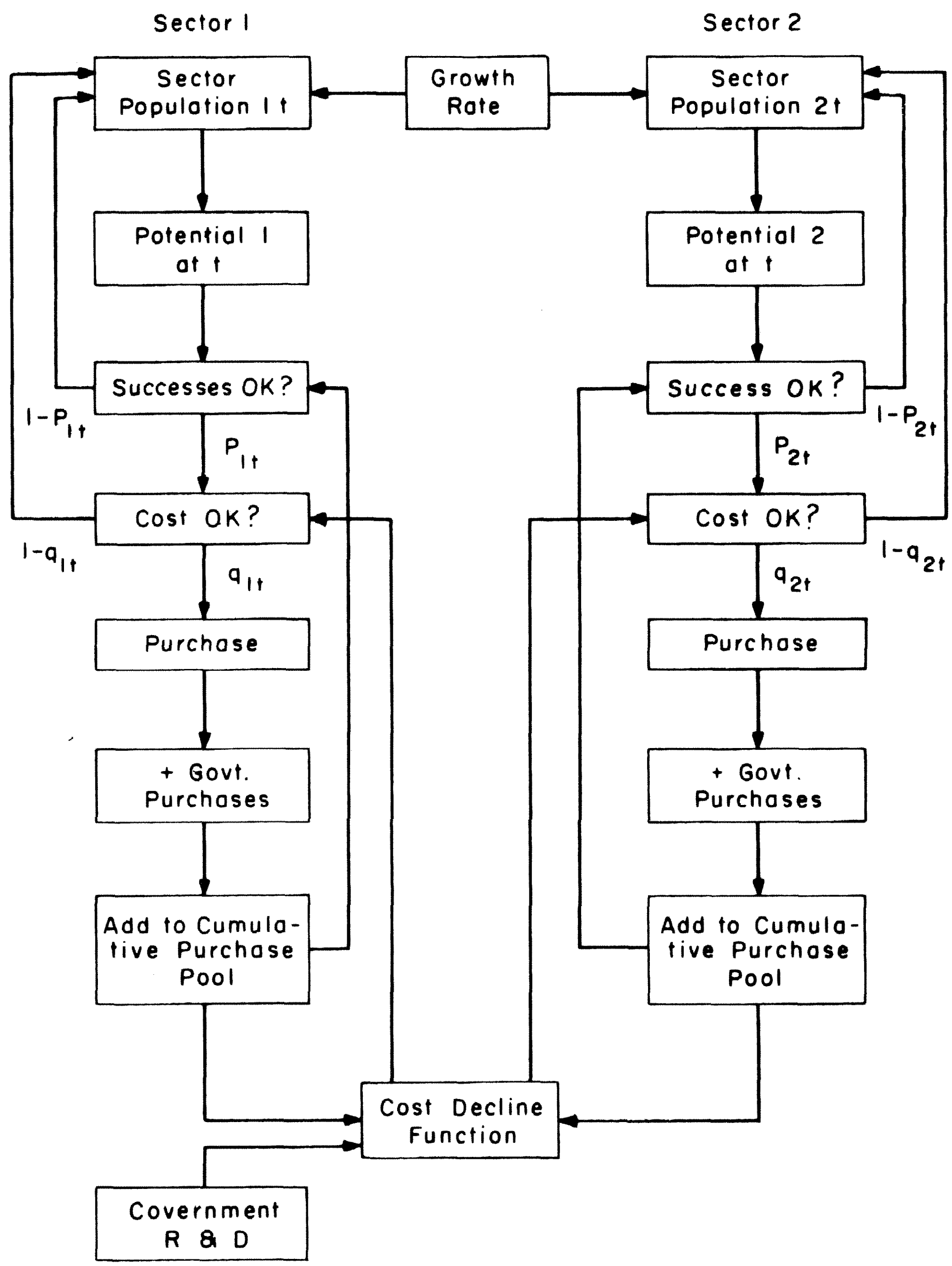
Cost decline results from either government R&D expenditures in process improvements, or from cumulative production (experience) and may be selected from among three alternatives in the model:

1. No cost decline—costs are unaffected by R&D or experience.
2. Exponential cost decline—the user specifies a level of R&D or production that would cause a cost reduction of 50%. This information is then used to construct exponential R&D or production cost-decline functions.
3. Arbitrary cost-decline function—the user may specify an arbitrary set of cost decline rates. A piecewise linear approximation to those decline-rates will be included in the model.

Since the cost of production is not highly dependent on application, the cost decline function is assumed to be common to sectors, that is, marketwide. Influences such as competing technologies are not considered in this model; rather, a few key variables are included to give a first approximation of likely market response.

Early in the life of a product, the only dynamic input to the decision process is government investment. By acting as a large, guaranteed consumer through pilot projects in various sectors, the government increases installations, thereby increasing likelihood of adoption in those sectors, and causes a cost decline through increased production. Figure 2 outlines the decision process in a model with two sectors. By providing funds for R&D,

Fig. 2. PVO model outline.



the government directly contributes to the reduction of cost, but not to the perceived effectiveness of PV technologies.

The PVO model is discrete in time and deterministic. When PVO models increase in installations, the unit cost decreases, over time. All variables related to number of installations are split into private and government parts. The variables that we want to follow are

X_{it} = number of kilowatts of government PV installations in sector I at time t,

Y_{it} = number of kilowatts of private PV installations in sector i at time t,

Z_i = size, in kilowatts, of a PV installation in sector i, and

C_t = cost per kilowatt of PV at time t.

Let

$N_{it} = \sum_{\tau=0}^{t-1} (S_{i\tau} + Y_{i\tau})$ be cumulative kilowatts installed in sector i before t and

$N_t = \sum_i N_{it}$ be cumulative kilowatts across sectors.

A standard form for a cost decline is "constant doubling," where cost is discounted by a fraction λ when production doubles, or (where C_0 is initial cost):

$$C_t = C_0 \, (N_t/N_0)^{\log_2 \lambda}.$$

As noted earlier, other more general cost-decline forms may be included in the model.

The question is thus at the next time step how many additional square feet will be bought as a function of this cost. Our assumptions imply that the fraction of consumers who will buy are those who find the cost low enough and the number of prior successful installations high enough (subject to their perception of PV). This we model as follows:

$$\int_{C_i}^{\infty} f_{C_i}(p)\, dp = 1 - F_{C_i}(C_t),$$

the probability that C_t is acceptable, and

$$\int_{-\infty}^{N_{it}/Z_i} f_{S_i}(p)\, dp = F_{S_i},$$

the probability that N_{it}/Z_i successes are acceptable in sector i. To model the potential market, define

P_{io} = initial potential existing installations in sector i,

Q_i = number of potential original equipment installations per time period, and

g_i = growth rate of old potential installations.

We distinguish between "old" and "new" potential installations so we can model sectors that discriminate between retrofit and original equipment installation in varying degrees with a single functional form. Then total market potential in sector i at time t is

$$P_{it} = (P_{it-1} - Q_i)\,(1 + g_i) + Q_i.$$

Before time t, N_{it}/Z_i installations have already been made. The remaining private purchase potential is decreased by three factors. The two acceptability criteria of cost and successes have already been discussed. Given acceptance on that basis, there is then a product-perception/probability of choice: $h(\mathbf{A})$, where $\mathbf{A}$ is a vector of product/market characteristics. Thus the number of private square feet of PV purchases at time t in sector i is expected to be

$$Y_{it} = (P_{it} - N_{it-1}/Z_i - X_{it}/Z_i)[1 - F_{C_i}(C_t)]F_{S_i}(N_{it}/Z_i)\,h(\mathbf{A})\,Z_i.$$

Now that we have Y_{it} we can find N_{it} and N_t, setting the stage for advancing to $t + 1$.

We can use this model to formulate a simple decision problem for the government. The government's problem is to decide how much to spend and how to allocate demonstration project resources. This becomes

find	$\{X_{it} \text{ and } R_t\}$
to maximize	$\sum_i \sum_t Y_{it}$
subject to	$\sum_i X_{it} C_t + R_t \leq B_t$, for all t,
where	Z_t = annual government budget constraint
	R_t = amount of R&D investment at time t.

(A later version of PVO will incorporate subsidies as well.)

PVO has been implemented as a system of computer programs and can currently be run in simulation mode (i.e., to evaluate a given plan), or in optimization mode. The use of the model in optimization mode requires that a good starting point be found; the previous analysis with simple models is used to generate such points.

MODEL CALIBRATION

A key aspect of the government's PV program is to gather data about likely sector response to photovoltaics. This has been done through surveys gathered at experimental field installations. Three such experiments have been run:

1. 296 interviews with farmers at an experimental installation in Mead, Nebraska, in July 1977,
2. 252 interviews at a PV demonstration at the Nebraska State Fair in September 1977, and
3. 226 interviews with current and prospective homeowners in Boston on Sun Day, May 1978.

The results of those analyses are reported in detail elsewhere [10, 11]. To summarize our findings, we have found the following for the agricultural sector:

Only three–four demonstration projects are needed to eliminate the product risk perception among farmers.

Exposure to a working PV site makes farmers more aware of potential energy savings than does a description of the system.

Key factors associated with PV adoption are
newness/expense,
complexity of system and use of untried concepts, and
independence from traditional fuel sources.

Exposure to a PV demonstration site affects the way farmers think about irrigation but does not affect their preference for the system. (This means that a carefully designed *advertising* program can have the same effect on system preference as would a demonstration program).

Similarly, for Nebraska consumers, we saw the following:

> Exposure to a PV site alleviates the need for a respondent to gain expert approval before accepting PV as an alternative to traditional electrical systems.
>
> Exposure to a PV site increases a respondent's preference for photovoltaics.
>
> Exposure to an energy independent concept brought out concerns about the reliability of the system.
>
> Exposure to a utility-dependent concept brought out more ecologically oriented concerns.
>
> Key factors associated with PV preference and perceptions are
> complexity/untried concepts,
> reliability/safety, and
> pollution reduction/energy conservation.

Finally, our Sun Day analysis showed the following:

> Three perceptual factors were identified in this study, with the first being by far the most important in determining photovoltaic preference:
> economical/ecological soundness,
> complexity/untried concepts, and
> secondary benefits.
>
> These factors explain photovoltaic preference well.
>
> Very little difference exists between subpopulations whether broken down by current/prospective homeowner or by utility-dependent/independent concepts. (Half the sample was told PV would make them energy-independent; the other half was told they would stay connected to the grid.)

There is a big difference between the Sun Day results and the corresponding Nebraska results. The Sun Day population better accepts the technological feasibility of photovoltaics and is more concerned with its noneconomic benefits. But this difference is not reflected in any of the available demographics.

POLICY ANALYSIS

A series of model runs was made with the PVO model for three sectors: agriculture, residential, and central power. The field measurements previously described were used to develop meaningful model coefficients for agricultural and residential sectors, especially to parametrize the cost and successful installations distributions. We must not only know the probability of purchase given acceptance, but also how this number changes with customer attitudes. Our model allows us to run sensitivity analysis on various effects of marketplace changes.

Within the residential and agricultural sectors we have created eight regions that can be thought of as separate geographical subsectors. These subsectors were defined to be of equal size and have the same market growth rates and cost acceptability criteria.

Our initial test of the PV diffusion model assessed the impact of different allocation strategies of government funding on the diffusion of PV technology. At the time of study, the government was proposing to spend $286 million over the next eight years on PV market development. The purpose of this analysis was to investigate the sensitivity of market response to varying allocation strategies. Of interest are the following results.

1. The empirical acceptance curves were S-shaped. In accordance with this result, in any sector, the optimal allocation was to "build up to a point and then go on." An improvement of 273% over a "concentrate in one geographic region strategy" was seen here.
2. The level of government demonstration program resources is greater than that needed to begin the diffusion in all the sectors. Thus, again in accordance with our S-shaped results, allocation to all three sectors did 28% better than the best single-sector strategy.
3. Overall, the best strategy, by another 2%, is to allocate to agriculture first, region by region, then to residential, and then to central power. Although this is in inverse order of size of market, it is directly related to the rule developed in the proof for Theorem 4: PV is most cost effective for agriculture, least so for central power.
4. All optimal policies build up to the budget constraint in each year, consistent with our earlier observations about not delaying demonstration investment.
5. The sensitivity of total market demand to the rate of cost decline is dramatic. Increasing the learning curve factor from 0.70 to 0.75 decreases total peak watts purchased by 95.5%. The sensitivity of market demand to the learning factor leads to two conclusions: a) unless cost declines rapidly over time (i.e., by a factor greater or equal to 30% for every doubling of output), the proposed level of funding is insufficient to meet the DOEs price and market goals; b) a detailed study of the supply curve must be completed prior to beginning an analysis of the benefits of a government procurement program.
6. The impact of a 12% improvement in consumer attitudes toward PV was-evaluated on one of two questions: either the cost/benefit or sensitivity to weather damage question. (This could be achieved through a government or private sector communication program.) This change increases total wattage installed by 13.1%, decreases end-period price by 6.1%, and increases private investment by 11.1%.

These results are interesting: They point to a general consistency and interrelationship between the simple, analytical models analyzed earlier and a more complex operational model such as PVO.

Conclusions

As Mahajan and Peterson [13] note, we do not have a valid theory of innovation diffusion. Yet, the models reviewed here capture some of the flavor of the diffusion process and are useful descriptive tools. Our results in the second and third sections give some limited insight into the policy implications of these models. And, as the fourth section shows, these results help us to get more complex models started and to explain the results of those models.

We still need a more general, valid theory and set of models of innovation diffusion (see [9] for a review). We must make use of what we have, however, building and analyzing simple models to gain insight and more complicated models to guide policy development.

Appendix: Proofs of Theorems

PROOF OF THEOREM 1

If d is concave, so is $2p + d(A_1) + d(A_2)$ and so is the Lagrangian

$$\mathcal{L} = 2p + d(A_1) + d(A_2) + \lambda(A_1 + A_2 - K).$$

The first-order conditions imply that

$$\frac{\partial \mathcal{L}}{\partial A_i} = d'(A_i) + \lambda = 0, \qquad i = 1, 2,$$

or $d'(A_i) = d'(A_2)$ and $A_1 = A_2 = \frac{1}{2}K$ by the concavity of d. By the concavity of $\mathcal{L}$, this solution is a maximum and a global optimum. The extension to N sectors is straightforward.

Our conclusion is that if the imitation parameter is a concave function of the number of demonstration installations, the optimal allocation is an equal or spread-out development policy.

PROOF OF THEOREM 2

If d is convex, consider $F = 2p + d(A_1) + d(A_2)$, which is also convex.

Over the region $A_1 + A_2 \leqslant K$ (with $A_1, A_2 \geqslant 0$), F must then achieve its maximum at an extreme point, either $A_1 = K$ or $A_2 = K$. Clearly, any intermediate policy (A_1^*, A_2^*), $A_1^* > A_2^*$, can be improved by a small amount to $(A_1 + \Delta, A_2 - \Delta)$ since $d(A_1 + \Delta) - d(A_1) > d(A_2) - d(A_2 - \Delta)$ by the convexity of d. Thus, as any intermediate solution can always be improved, the optimum must occur at an extreme point.

PROOF OF THEOREM 3

If d_i is concave, the Lagrangian,

$$\mathcal{L} = \sum p_i S_i + \sum d_i(A_i) S_i - \lambda(K - \sum A_i)$$

is also concave. Thus, a unique, global solution is obtained by solving

$$\frac{d\mathcal{L}}{dA_i} = 0 = S_i d'_i(A_i) = \lambda \quad \text{for all } i \qquad \text{and} \quad K = \sum A_i.$$

The solution here is to find the A_i so that equal marginal innovator returns $\big(S_i d_i'(A_i)\big)$ are obtained for each region or sector.

PROOF OF THEOREM 4

The argument here is the same as that in Theorem 2. The market to allocate resources to is market i such that $S_i d_i'(K)$ is maximal.

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