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A UTILITY MODEL FOR PRODUCT POSITIONING

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ABSTRACT

A model and procedure are proposed to help design and position products which are characterized by a high level of consistency between product preference and purchase behavior. The procedure is based on utility theoretic concepts for assessing preference and inferring probable behavior. A numerical example is included.

Firms continually face positioning and design issues related to their products. The product design issue is essentially: "What physical and psychological characteristics would I like my (new or revamped) product to have either to maximize profit, market share, or more generally, the firm's expected utility?" This product design question is relevant to both new and existing products. The characteristics that can be controlled include price, several dimensions of use or quality, packaging, flavor (perhaps), etc. The firm generally has explicit control over physical quantities (price, taste, objective performance) and implicit control, through message and communications design, over the psychological quantities (a young-swinging beer, e.g.).

How should products be designed and how can those designs be changed during a product's life cycle to meet a firm's objectives? There have been two modes of analysis to help answer these questions. One approach is that of the psychometricians. In particular, making use of similarity data, Stefflre [13] developed "perceptual maps" of market structures. He suggested introducing brands in areas of the map which were relatively vacant. Other work has followed; a summary of the multidimensional scaling literature is available in Green and Carmone [4] and Green and Rao [5].

Another approach to the design question is provided by marketing model builders. Many attempts have been made to establish functional forms which relate product attributes, such as price, to output measures such as market share. Kotler [10] provides a review of much of the literature.

A more recent, promising approach is one taken by Urban [14]. Urban uses psychometric techniques to map the space of consumers' attitudes toward existing brands as well as an "ideal" brand. He postulates that the farther a brand is from the ideal point, the lower its market share should be. The procedure, which models the trial and repeat-purchase processes separately, assumes that a brand's long run market share is a parameterized quadratic function of its distance from the ideal point. That function is then calibrated using actual market share for a number of new brands quite well. But, due to data requirements, the model's use would seem to be limited to frequently purchased products. Virtually all work in these areas has been limited to frequently purchased products, mainly due to the greater availability of data. Thus, the problem of analytically positioning consumer durable products and most industrial products has been largely ignored.

An important difference between consumers purchasing packaged foods, and companies purchasing fabricated materials is that such industrial purchasing behavior conforms more closely to prescribed criteria (see Webster and Wind [15]). Thus one may "prefer" Krispy Krakers to "Whole Oats" and purchase Whole Oats anyway due to "variety seeking," availability, or other random behavior. The same inconsistency is not likely to occur in the purchase of hydrochloric acid, where price and delivery terms are preeminent, or in the selection by parents of a medical treatment for a baby's congenital defect.

The methodology suggested is designed to be used for precisely those situations in which little data (of a repeat-purchase variety) are likely to be available, but where the customers (individuals or firms) purchase consistently with stated or inferred preferences. The approach is based on utility theory (see Raiffa [11] for a discussion of the basic concepts of utility theory). It assumes customers have a von Neumann-Morgenstern utility function defined over the product variables -- that is, customers are expected utility maximizers and "act as they should." Utility theory has been used in the past mainly in a normative or prescriptive sense -- telling decision makers what they should do in given circumstances. The market situations

considered here are, by definition, those in which individuals do what they should.

Hauser and Urban [6] have developed a structure for models of choice between finite alternatives. Their structure of the analytical process of choice includes: (1) observation of behavior and measurement; (2) reduction and abstraction -- reducing the number of product dimensions to a few, "independent" ones and labelling them; (3) compaction, developing brand preference measures; (4) probability of choice, relating preference to purchase behavior, and (5) aggregation, transforming probability of purchase measures to market share measures.

The methods developed here suggest augmenting the observation step, by measuring attitudes in face-to-face interviews, through methods of direct utility function assessment as in [1], [8], [9]. The compaction operation then uses utility theory and the assumption of a von Neumann-Morgenstern utility function obviates the probability-of-choice step. Aggregation is performed by taking explicit account of consumer heterogeneity throughout the procedure.

This procedure would be useful in industrial purchasing situations where the purchasing agent or a suitable surrogate could be isolated which has both explicit purchase criteria and bargaining power. The purchase of graded goods (raw materials, fabricated materials, and supplies) would be a common example.

The paper is organized as follows: Section 1 presents the formal structure of the model and introduces notation. Section 2 develops a general framework of analysis which is applied to a simple, hypothetical example in Section 3.

## 1. THE MODEL AND NOTATION

Consider a well-defined product class in  $N$  firms  $F_1, \dots, F_N$  in a single, specific market. Let  $m_i$  be the market share of  $F_i$ , and assume that each firm makes a single product.

Let the set of attributes  $X_1, X_2, \dots$  completely characterize a product, where  $X_1$  could be price,  $X_2$  could be reliability ratings, etc. These attributes would be attained by factor analysis of a series of well-defined product ratings or perhaps non-metric scaling procedures, given a set of similarity judgments. Both methods have proven effective through neither has established "superiority." (Green, [3]) The output of these procedures, then, would be a reduced set of product characteristics,  $X_1, \dots, X_J$ . A specific level of  $X_j$  is  $x_j$  so a product

is completely described by  $\underline{x} = (x_1, \dots, x_J)$ . The product of firm  $F_i$  will be denoted by  $\underline{x}^i = (x_1^i, \dots, x_J^i)$ . A no-product purchase  $\underline{x}^0 = (x_1^0, \dots, x_J^0)$  could be included for completeness.

Customers will be designated by  $C_1, \dots, C_K$ . It will be assumed that each customer  $C_K$  has a von Neumann-Morgenstern utility function  $u_K(\underline{x}/\underline{\lambda})$ , where  $C_K$ 's utility function is specified by the set of parameters  $\underline{\lambda} = (\lambda_1, \dots, \lambda_R)$ . Assuming each customer buys a product, utility theory suggests he should buy the product of firm  $F_i$  such that his utility is maximized.

Since viewing the problem from the firm's point of view will require the same methodology regardless of the specific firm, let us take the viewpoint of firm  $F_1$ . Firm  $F_1$  has certain objectives which could include maximizing market share, maximizing profit, and so on. We postulate that the objective function of  $F_1$  is also specified by a von Neumann-Morgenstern utility function  $v_1$  over market share  $m_1$ , profit and/or other variables.

There are uncertainties here for both firms and customers. The firm wants to know utility functions for all customers in the market. This information about customer heterogeneity will be expressed in the form of a probability distribution  $P_\lambda(\underline{\lambda})$  over the parameters  $\underline{\lambda}$  describing a randomly selected customer's utility function. Thus, customers' utility functions are likely to differ so  $\underline{\lambda}$  does not take on a single value, but, rather, is expressed as a probability distribution. Any firm will not have perfect knowledge about the "true"  $P_\lambda(\underline{\lambda})$  and will, in general, attempt to estimate the distribution, entailing some error. Thus, we might consider parameterizing the distribution to give  $P_{\lambda/\theta}(\underline{\lambda}/\underline{\theta})$  where  $\underline{\theta} = (\theta_1, \dots, \theta_T)$  indicates the uncertainties of  $F_1$  about the true  $\underline{\lambda}$ . We quantify that uncertainty by the probability distribution  $P_\theta(\underline{\theta})$ .

Consumers in general will differ in their knowledge or attitude about the characteristics  $\underline{x}^i$  of firm  $F_i$ 's product. From  $F_1$ 's perspective, this heterogeneity can be described by the parameterized probability distributions  $P_X^i(\underline{x}^i/\underline{\phi}^i)$ ,  $i=1, \dots, N$ , where  $\underline{\phi}^i$  indicates a set of parameters for firm  $F_i$ . Again  $F_1$  may be unable to estimate these distributions without error, so uncertainty will exist which we quantify as  $P_\phi(\underline{\phi})$ , where  $\underline{\phi} = (\phi^1, \phi^2, \dots, \phi^N)$ .

To summarize here, our model contains (from  $F_1$ 's view):

1. An objective function  $v_1$ , which is a utility function, known with certainty.

2. A distribution of utility functions  $u_k(\underline{x}/\underline{\lambda})$  which vary across the heterogeneous customer population quantified by  $P_{\lambda/\theta}(\underline{\lambda}/\underline{\theta})$ . The probability distribution  $P_{\theta}(\underline{\theta})$  quantifies this uncertainty.
3. A set of distributions of product-perceptions,  $P_X^i(\underline{x}^i/\underline{\phi}^i)$ ,  $i=1, \dots, N$ , also varying across the heterogeneous customer-population. The probability distribution  $P_{\phi}(\underline{\phi})$  quantifies this uncertainty.

## 2. GENERAL MODEL STRUCTURE

In this section, we first consider the decision an individual consumer must make and how his decisions are inputs to the decision-making processes of the firm. Then we focus on how firms can use the model for product positioning decisions.

### 2.1 Consumer Decisions

The consumer must decide which if any of the products in the market to buy given that he will buy at most one. Thus we explicitly consider the case of a consumer not purchasing any product. Another possibility would have been to define consumers as those who will buy one product and then formally include uncertainty about the number of consumers in the market.

Our model does not explicitly include individual consumer uncertainty about product characteristics. Rather,  $F_i$  is uncertain both about the set of consumer utility functions and the set of consumer product perceptions. Explicit inclusion of consumer uncertainty would needlessly complicate the problem. We will assume that  $C_k$  has a utility function  $u_k(\underline{x})$  and his choices are not to buy a product and receive  $\underline{x}^0$  or to buy the product of  $F_i$  and receive  $\underline{x}^i$ ,  $i=1, 2, \dots, N$ . He will choose the option  $\underline{x}^*$  giving him the highest utility where  $\underline{x}^*$  is defined by

$$(1) \quad u_k(\underline{x}^*) = \max_{i=0, 1, \dots, N} \{u_k(\underline{x}^i)\}$$

In the case of uncertainty, the consumer should choose the product of firm  $F_i$  which maximizes his expected utility. If  $P_X^{ik}(\underline{x})$  represents the judgment of consumer  $C_k$  about firm  $F_i$ 's product, the option  $\underline{x}^*$  should be chosen such that

$$E[u_k(\underline{x}^*)] = \max_{i=0,1,\dots,N} \int u_k(\underline{x}) P_X^{ik}(\underline{x}^i) d\underline{x}^i,$$

where  $E[u_k(\underline{x}^*)]$  is the expected utility of the product of firm  $F_1$  (where  $F_0$  designates the no product option) for individual  $C_k$ .

## 2.2 Firm Decisions Under Certainty

Under certainty, firm  $F_1$  should maximize its market utility  $v_1$ . Here we assume the distribution of utility functions is known and that customers do not vary in their perception of brand characteristics (that is  $\theta$  and  $\phi^i$ ,  $i=1, 2, \dots, N$ , are known). The condition of certainty could be used as a first cut, as less information is required here to attain a product design decision. Firm  $F_1$  has a product with characteristics  $\underline{x}^1$  and the population utility functions are represented by  $u_k(\underline{x}/\lambda)$  where  $P_\lambda(\lambda)$  represents the population heterogeneity. From (1) it follows that a consumer with parameters  $\lambda$  will choose firm  $F_1$ 's product if

$$(2) \quad u_k(\underline{x}^1/\lambda) > u_k(\underline{x}^h/\lambda), \quad h=0,1,\dots, N, \quad h \neq 1,$$

where we neglect ties in terms of equally desirable alternatives. The proportion of consumers  $m_1$  who choose  $\underline{x}^1$  is

$$(3) \quad m_1 = \int_{\lambda \in \Lambda_1} P_\lambda(\lambda) d\lambda,$$

where  $\Lambda_1$  is that set of  $\lambda$ 's such that (2) holds. Neglecting ties,  $\Lambda_0, \Lambda_1, \dots, \Lambda_N$  are mutually exclusive and collectively exhaustive so  $\sum m_i = 1$  as required.

Suppose  $F_1$  is considering changing its product position from  $\underline{x}^1$  to  $\hat{\underline{x}}^1$ . Then a new  $\hat{m}_1$  can be determined exactly as  $m_1$  was using (2) and (3).

Suppose a new firm with utility function  $v$  is trying to enter a volume inelastic market with a maximum profit product ( $\underline{x}$ ); that is, say  $v = m \cdot G \cdot (s - C(\underline{x})) - d(\underline{x})$  where  $s$  is unit product selling price,  $C$  is the unit cost of the product ( $\underline{x}$ ),  $G$  is the market volume,  $m$  is its market share, and  $d(\underline{x})$  is the fixed development cost (plant, R&D, etc.) associated with ( $\underline{x}$ ). Then  $v$  is the profit associated with the new product. There will likely be a set  $Q$  of alternate product positions, generically denoted by  $\underline{x}$ , that the new firm could attain.

Given the existing products in the market, for each possible  $\underline{x}$  there is a set  $\Lambda(\underline{x})$  (perhaps null) defined by

$$(4) \quad \Lambda(\underline{x}) \equiv \{\lambda: u_k(\underline{x}/\lambda) > u_k(\underline{x}^i/\lambda), i=0, 1, \dots, N\}$$

The best decision for this firm is to choose  $\underline{x}$  in  $Q$  to maximize

$$(5) \quad v(\underline{x}) = \left[ \int_{\lambda \in \Lambda(\underline{x})} P_\lambda(\lambda) d\lambda \right] \cdot \left[ (G \cdot (s-C(\underline{x})) - d(\underline{x})) \right]$$

where the first term in brackets in (5) represents the market share of product  $(\underline{x})$  and  $v(\underline{x})$  is the profit associated with the product. Under "nice" conditions it may be possible to simply differentiate  $v(\underline{x})$  with respect to  $\underline{x}$  to determine the product position to maximize profit.

### 2.3 Firm Decisions Under Uncertainty

The problems under uncertainty are parallel but more complicated than those with certainty. The two sources of uncertainty are the firms' imperfect knowledge about preference (or utility) heterogeneity, characterized by  $\underline{\theta}$ , and imperfect knowledge about perceptual heterogeneity, characterized by  $\underline{\phi}^i$ ,  $i=0,1,\dots,N$ .

We will assume here, for simplicity, that perceptual heterogeneity and utility heterogeneity are independent within individuals. Fix the utility heterogeneity parameters as  $\underline{\lambda}$  and select a customer with utility function  $u(\underline{x}/\underline{\lambda})$  at random. That customer's selection will be from the set of product characteristic distributions  $P_X^i(\underline{x}^i/\underline{\phi}^i)$ ,  $i=0,1,\dots,N$ . The product purchased should be  $\underline{x}^*$  such that

$$E[u(\underline{x}^*/\underline{\lambda}, \underline{\phi})] = \max_{i=0,1,\dots,N} E[u(\underline{x}^i/\underline{\lambda}, \underline{\phi}^i)]$$

where  $E[u(\underline{x}^i/\underline{\lambda}, \underline{\phi}^i)]$  indicates the expected utility using  $u(\underline{x}/\underline{\lambda})$  of  $\underline{x}^i$  given  $\underline{\lambda}$  and  $\underline{\phi}^i$ . That is,

$$(6) \quad E[u(\underline{x}^i/\underline{\lambda}, \underline{\phi}^i)] = \int_{\underline{x}^i} u(\underline{x}^i/\underline{\lambda}) P_X^i(\underline{x}^i/\underline{\phi}^i) d\underline{x}^i.$$

As before, define  $\Lambda_i(\underline{\phi})$  as the set of  $\lambda$  for which  $\underline{x}^* = \underline{x}^i$ , where  $\Lambda$  is now dependent on  $\underline{\phi}$ . The expected market share of firm  $F_i$ , given  $\underline{\theta}$  and  $\underline{\phi}^i$ ,  $i=1,2,\dots,N$ , are known, is

$$(7) \quad m_i(\underline{\theta}, \underline{\phi}) = \int_{\lambda \in \Lambda_i(\underline{\phi})} P_{\lambda/\theta}(\lambda/\underline{\phi}) d\underline{\lambda}.$$

If  $F_1$ 's utility function  $v_1$  has market share as its argument, it should choose  $\underline{x}^i$  to maximize its expected utility, given by

$$(8) \quad E(v_1) = \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\phi}}^{\bar{\phi}} v_1[m_1(\underline{\theta}, \underline{\phi})] P_{\theta}(\underline{\theta}) P_{\phi}(\underline{\phi}) d\underline{\phi} d\underline{\theta}$$

where  $P_{\theta}$ ,  $P_{\phi}$  describe the uncertainty in  $F_1$ 's measure of consumer's preferences and perceptions respectively.

### 3. A SIMPLE EXAMPLE (PRICE INPUTED QUALITY)

A number of authors (see Rao and Shakun [12], Gabor and Granger [2], Kamen and Toman [7]) have suggested that, in certain product classes which offer nearly indistinguishable products, (such as gasoline, packaged soaps, etc.) price, as an indicator of quality (and perhaps value) is the most important, if not sole determinant of purchase behavior. Without delving into this subject we offer the following simple example in which a single product-characteristic (say, price) distinguishes market products. Thus, the product characteristics are described by the single attribute  $X$  (price) and  $\lambda$ ,  $\theta$ ,  $\phi^i$ , for all  $i$ , are univariate. The general problem is tractable with aid of a computer, perhaps through simulation, if necessary, but a more complex computer, perhaps through simulation, if necessary, but a more complex example here would obscure the basic ideas of the method.

Let the set of utility functions of the consumers be

$$(10) \quad u_k(x/\lambda) = \lambda x - x^2, \quad k = 1, \dots, K.$$

where  $x$  is a price for the product and suppose the "true" distribution of  $\lambda$  among consumers is quantified by

$$(11) \quad P_{\lambda}(\lambda) = \frac{1}{10}, \quad 0 \leq \lambda \leq 10.$$

There are three firms  $F_1$ ,  $F_2$ , and  $F_3$  and the consumers are heterogeneous in their perceptions of prices  $x^1$ ,  $x^2$ , and  $x^3$  as follows:

$$(12) \quad P_X^i(x^i/\phi^i) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x^i - \phi^i)^2}{2}} \quad i = 1, 2, \dots$$

That is, the  $P_X^i$  are normal distributions with means  $\phi^1$ ,  $\phi^2$ , and  $\phi^3$  respectively and unit variances (see Gabor and Granger [2] for empirical justification using  $x_i = \log$  price).



### 3.1 Consumer Decisions

Given the uncertainty encoded by (12), the expected utility of product  $x^i$  to consumer  $C_k$  is

$$\begin{aligned}
 (13) \quad E[u_k(x^i/\lambda)] &= \int_x u_k(x/\lambda) P_X^i(x/\phi^i) dx \\
 &= \int_x (\lambda x - x^2) \frac{1}{2\pi} e^{-\frac{(x-\phi^i)^2}{2}} dx \\
 &= \lambda \phi^i - (\phi^i)^2 - 1,
 \end{aligned}$$

where clearly  $\phi^i$  is the mean value of  $x^i$ , or mean perceived price.

Let us assume that

$$(14) \quad \phi^1 = 2, \phi^2 = 3, \text{ and } \phi^3 = 5.$$

Now we want to find the values of  $\lambda$  for which  $x^1$ ,  $x^2$ , and  $x^3$  are preferred. From (13), it follows that  $x^1$  is preferred to  $x^2$  on average if and only if

$$\lambda \phi^1 - (\phi^1)^2 - 1 > \lambda \phi^2 - (\phi^2)^2 - 1$$

which holds if  $\lambda < \phi^1 + \phi^2$ . Similarly  $x^1$  is preferred to  $x^2$  if and only if  $\lambda < \phi^1 + \phi^3$  and  $x^2$  is preferred to  $x^3$  if and only if  $\lambda < \phi^2 + \phi^3$ . From this, since  $\phi^1 < \phi^2 < \phi^3$ , we have

$$(15) \quad \left. \begin{array}{l} x^1 \\ x^2 \\ x^3 \end{array} \right\} \text{ preferred iff } \left\{ \begin{array}{l} \lambda < \phi^1 + \phi^2 = 5 \\ \phi^1 + \phi^2 \leq \lambda < \phi^2 + \phi^3 \\ \lambda \geq \phi^2 + \phi^3 = 8 \end{array} \right.$$

The market share of firm  $F_1$  should be

$$(16) \quad m_1 = \int_{\lambda=0}^5 P_\lambda(\lambda) d\lambda = \int_{\lambda=0}^5 \frac{1}{10} d\lambda = \frac{1}{2}.$$

In an analogous fashion,  $m_2 = \frac{3}{10}$  and  $m_3 = \frac{2}{10}$ .

### 3.2 Firm Decision Under Certainty

Let us take a look at one special problem here. Suppose a new firm  $F_4$  were to introduce a new product to compete with  $x^1$ ,  $x^2$ , and  $x^3$ . If the consumer distribution of  $x^4$  is to be characterized by  $\phi^4$  and

$$(17) \quad P_X^4(x/\phi^4) = \frac{1}{2\pi} e^{-\frac{(x - \phi^4)^2}{2}},$$

what is the best value of  $\phi^4$  (mean perceived price) to maximize  $m_4$ , the market share of firm  $F_4$ . (We are assuming firm  $F_4$  accepts the other information in the problem, e.g. the  $P^i(x/\phi^i)$ ). From analysis similar to that leading to (15), it follows that if  $\phi^4 < \phi^1$ ,

$$\left. \begin{array}{l} x^4 \\ x^1 \\ x^2 \\ x^3 \end{array} \right\} \text{ preferred iff } \left\{ \begin{array}{l} \lambda < \phi^4 + \phi^1 \\ \phi^4 + \phi^1 < \lambda \leq \phi^1 + \phi^2 \\ \phi^1 + \phi^2 < \lambda \leq \phi^2 + \phi^3 \\ \lambda \geq \phi^2 + \phi^3 \end{array} \right.$$

so  $m_4 = \frac{\phi^4 + \phi^1}{10}$ . If  $\phi^1 < \phi^4 < \phi^2$ , then

$$m_4 = \frac{\phi^4 + \phi^2 - (\phi^4 + \phi^1)}{10} = \frac{\phi^2 - \phi^1}{10}; \text{ if } \phi^2 < \phi^4 < \phi^3, \text{ then}$$

$m_4 = \frac{\phi^3 - \phi^2}{10}$ ; and if  $\phi^4 > \phi^3$ ,  $m_4 = 0$  since  $x^4$  would only be preferred if  $\lambda > \phi^4 + \phi^3$  and  $\phi^3 = 5$  and  $\lambda_{\max} = 10$ . Thus, it is clear

that under certain knowledge on the part of the firm  $F_4$ , the optimal  $\phi^4$  subject to (17) is  $\phi^4 + \varepsilon = \phi^1 = 2$  for some small  $\varepsilon$ . Then  $m_4 = \frac{\phi^1 + \phi^4}{10} = \frac{4 - \varepsilon}{10}$ .

### 3.3 Firm Decisions under Uncertainty

Return now to the three firms with three products described by (12) and suppose the class of consumer utility functions is given by (10). Suppose that Firm  $F_1$ , whose point of view we will take, feels

$$(19) \quad f_\lambda(\lambda/\theta) = \frac{1}{\theta}, \quad 0 \leq \lambda \leq \theta,$$

but due to the lack of available data, the firm's market research team feels there is some uncertainty about the true  $\theta$ , which is characterized by

$$(20) \quad P_\theta(\theta) = \frac{1}{3}, \quad 9 \leq \theta \leq 12.$$

Suppose that  $x^2$  is the "standard product" in the market that everyone knows well and  $\phi^2 = 3$ . Our own product  $x^1$  and the competitor's

$x^3$  are newer so they are subject to more consumer variability. So let  $\phi^1$  be uniformly distributed from 1 to 3 and let  $\phi^3$  be uniformly distributed from 4 to 6. If these perceptions are independent, we have

$$(21) \quad P_{\phi}(\phi^1, \phi^2, \phi^3) = \frac{1}{4}, \quad 1 \leq \phi^1 \leq 3, \quad \phi^2 = 3, \\ 4 \leq \phi^3 \leq 6.$$

It is easy to see that, as before  $\phi^1 < \phi^2 < \phi^3$  so analogous to (15),

$$(22) \quad \left. \begin{matrix} x^1 \\ x^2 \\ x^3 \end{matrix} \right\} \text{ preferred iff } \begin{cases} \lambda < \phi^1 + \phi^2 \\ \phi^1 + \phi^2 < \lambda \leq \phi^2 + \phi^3 \\ \lambda \geq \phi^2 + \phi^3 \end{cases}$$

Hence to calculate the probability distribution  $m_1$  for  $F_1$ , we just calculate the probability  $\lambda < \phi^1 + \phi^2$ . Note that the distribution of  $\phi^1 + \phi^2$  is uniform from four to six so if  $\alpha \equiv \phi^1 + \phi^2$ ;

$$(23) \quad P_{\alpha}(\alpha) = \frac{1}{2}, \quad 4 \leq \alpha \leq 6.$$

Refer to Figure 1 which shows the possible combinations of  $\theta$  and  $\alpha$ . From (20) and (23), it is clear that  $p(\theta, \alpha) = \frac{1}{6}$ . Note that  $m_1 = \frac{\alpha}{\theta}$

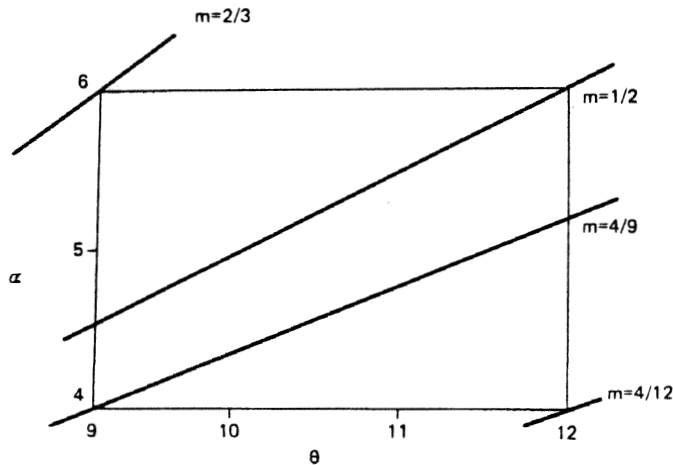


Figure 1

so at worst,  $m_1 = \frac{\alpha_{\min}}{\theta_{\max}} = \frac{1}{3}$  and at best  $m_1 = \frac{\alpha_{\max}}{\theta_{\min}} = \frac{2}{3}$ . Hence for  $\frac{1}{3} < m_1 < \frac{2}{3}$ , the probability  $m_1 < m$  equals  $\frac{1}{3} + \frac{1}{3} P(\frac{\alpha}{\theta} < m)$ . Integrating

over the appropriate regions of Figure 1, we get the cumulative probability distribution for  $m_1$  which is differentiated to yield

$$(24) \quad P_{m_1}(m) = \begin{cases} 0 & m_1 \leq \frac{1}{3} \\ 12 - \frac{4}{3m^2}, & \frac{1}{3} < m \leq \frac{1}{2} \\ \frac{21}{4} & \frac{4}{9} < m \leq \frac{1}{2} \\ \frac{3}{m^2} - \frac{27}{4} & \frac{1}{2} < m \leq \frac{2}{3} \\ 0 & m < \frac{2}{3} \end{cases}$$

This probability distribution is shown in Figure 2. The expected market share is given by

$$(25) \quad E[m_1] = \int_{\theta=9}^{12} \int_{\alpha=4}^6 \frac{\alpha}{\theta} \left(\frac{1}{6}\right) d\alpha d\theta = 0.466.$$

If firm  $F_1$ 's preferences are quantified with the utility function  $v_1$  over various market share levels, then one can simply use the probability distribution  $P_{m_1}(m)$  and  $v_1(m)$  to calculate the expected utility. Given a choice among options whose impacts are quantified by probability distributions over market share, firm  $F_1$  should calculate the respective expected utilities and choose the option associated with the highest.

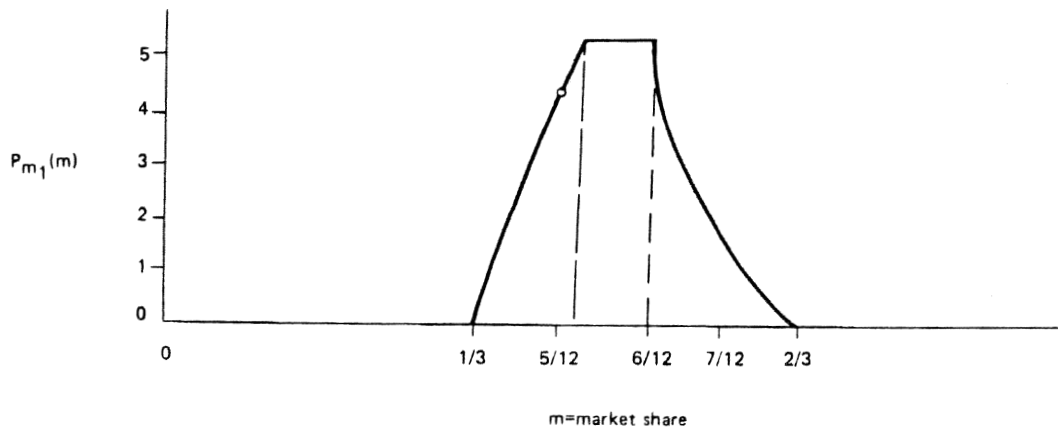


Figure 2

#### 4. MODEL STATUS AND USE

For a firm to utilize such a model, it would need to assess its utility function  $v$  and select parametric models for the population utility functions  $u_k(\underline{x}/\underline{\lambda})$ , their heterogeneity of preference  $P_{\lambda/\theta}(\underline{\lambda}/\underline{\theta})$ , and perceptions  $P_{\underline{x}}^i(\underline{x}^i/\underline{\phi}^i)$ ,  $i = 1, 2, \dots, N$ , of the products. A market analysis team can model these features, utilizing limited interviews with customers. Then the parameters  $\underline{\theta}$  and  $\underline{\phi}^i$   $i = 1, 2, \dots, N$ , could be quantified in a consumer survey. The relative frequency of responses could be smoothed to model the parameter distributions  $P_{\theta}(\underline{\theta})$  and  $P_{\phi}^i(\underline{\phi}^i)$ ,  $i = 1, 2, \dots, N$ .

As presented, the procedure is in a conceptual, proposal-for-use state. The work of Hauser and Urban [6], using some related concepts, has shown promising results. It is however too early to assess the applicability of this methodology.

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