MANAGEMENT SCIENCE
Vol. 20, No. 7, March, 1974
Printed in U.S.A.

A MODIFIED LINEAR LEARNING MODEL OF BUYER BEHAVIOR*

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A stochastic model of individual buyer behavior is developed from a set of postulates about the buying process. The postulates are shown to imply a linear learning model modified by a term to explain response to pricing stimuli. Thus, a customer's purchasing probability is modelled as a combination of the effect of his past purchasing behavior plus the effect of price-variation in the market. Methods are developed to calculate short- and long-term probabilistic properties of the process. A method for parameter estimation is included. The model differs from past modelling efforts in this area in that a controllable variable, product price, is explicitly included in the model-structure, allowing the model to be used to aid in pricing decision making under a certain set of assumptions about competitive behavior in a market situation.

I. Introduction

Most stochastic models of brand choice which have been suggested thus far have considered only time or past purchase feedback effects on future purchase probabilities. (Massy, Montgomery and Morrison [13] give a detailed review of this literature.) Thus, these models, while providing interesting insight into the purchasing process, are of somewhat limited value for decision making. The model developed here is an attempt to extend the power of such models by demonstrating (1) how a controllable marketing variable, product price, can be explicitly considered in the structure of the model and (2) how that model can then be used to aid in making pricing decisions. This paper will consider the theoretical development of the model; the performance of the model on live data and a set of illustrative numerical examples are the subject of a forthcoming publication.

In order to best explain the development of the model suggested here, we will briefly discuss a learning model first suggested by Bush and Mosteller [3] and applied to consumer behavior by Kuehn [10]. We will refer to Kuehn's model as the Simple Linear Learning Model (SLL) and to our Modified Linear Learning model as MLL. At the heart of SLL is the assumption that a customer's past-purchase tendency to buy a brand is a *linear function* of his prepurchase tendency. With respect to a particular brand, j, he can either buy j at time t, with associated probability $P_j(t)$ or a brand other than j with probability $1 - P_j(t)$. $P_j(t+1)$ is then related to $P_j(t)$ as follows:

$$(1)\quad \begin{aligned} P_j(t+1) &= \alpha + \beta + \lambda P_j(t) \quad &&\text{if } j \text{ is purchased at } t,\\ &= \alpha + \lambda P_j(t) &&\text{if } j \text{ is not purchased at } t. \end{aligned}$$

The model has at least one intuitively appealing aspect: the effect of past purchases on the present probability of purchase decreases geometrically. (A purchase K periods ago has weight λ^{K+1}, $\lambda < 1$, relative to the effect of the last purchase.) Thus, a customer's entire purchase history is considered, albeit in a highly structured way.

* Processed by Professor Donald G. Morrison, Departmental Editor for Marketing and Associate Editor J. Morgan Jones; received May 1973, revised August 1973.

This model makes certain important assumptions: it assumes that the parameters of the model do not change over time. It also assumes that all families exhibit adaptive behavior which can be described by linear feedback operators with approximately the same parameters (parameters are stationary across time and constant across families). This is a questionable assumption but not one which is overly restrictive, since to know $P_j(t)$ it is still necessary to know $P_j(0)$ as well as the customer's complete purchase history. Thus, this assumption does not restrict the richness (in terms of heterogeneity) of the model.

There are several difficulties with this model. Since the model is couched in terms of the probability of buying brand j, the probability of buying brands other than j must be updated after each purchase. It is not clear, however, that there is a uniquely logical way to do this updating. Another problem lies in the assumption that a purchase of a particular brand is *always* a reinforcement. This may be the case for homogeneous products but may be a weak assumption elsewhere. And this model (as well as most others in this class) ignores the effects of marketing stimuli or time related phenomena on brand choice probabilities.

A modification of this model which has decision-making implications is the subject of this paper.

II. Model Assumptions

Consider a rather homogeneous product class. Examples include gasoline, packaged soaps and detergents, cigarettes, etc., where it is intuitively acceptable (and in many cases, empirically verified) that actual brand-to-brand differences are very small although *perceived* differences may be large.

Suppose that the market breaks down logically into two mutually exclusive and exhaustive subsets—"Premium" (higher priced) brands and "Standard" (lower priced) brands. In many cases this distinction can be "store brand" vs. "national label," "majors" vs. "independents" (in gasoline), etc. Clearly, this model will be most useful where this distinction is most pronounced. In these very specialized market-types, two distinct forms of competition can occur: the Premium brands as a group can compete, by aggressive pricing, with the Standard brands for customers, or the Premium (or Standard) brands can compete among themselves for customers. If we choose to focus on price-promotions only (surely a good choice for the single most important promotion) then it is not illogical to treat these two groups as single brands and explore market behavior as if only two brands existed in the market.

A potentially complicated question arises: how can we simply represent the variation in price of the m (say) Premium and n Standard brands? However, in the markets we are considering this is frequently not a problem. Characteristically, gasoline markets (for example) have "major brand" price leaders and "independent brand" price leaders who set prevailing prices in a given market, though prices may change as frequently as daily.

In other words, price shifting within the Premium or Standard group is normally small relative to between group shifts, and we will choose to neglect it. Then, a prevailing Premium price and prevailing Standard price can be taken as (loosely) sufficient statistics for the brand prices in that market. It would thus seem reasonable under these circumstances, if one is interested in gains (or losses) to a particular "Premium" brand, to allocate sales changes in the entire group proportionally to some initial share-of-market measure.

Finally, we can reasonably argue that in the situation presented promotional effects other than price can be neglected: if we hypothesize that other promotions are geared mainly for Premium-Premium or Standard-Standard competition, then for Premium-Standard competition, Premium and Standard pricing will be the sole promotion of interest.

Briefly, then, we have set up a hypothetical situation in which some objections to SLL (i.e. that it does not consider market effects) can be met, and we have suggested some practical situations in which this type of behavior might be expected to hold.

Let us explicitly summarize the above discussion in a series of assumptions about the operation of the process which we will be exploring:

A1. The market breaks down naturally into two sets of brands, "Premium" and "Standard."

A2. The Premium brands can be characterized by a single price at a given time; the Standard brands can also be characterized by a single (generally lower than Premium) price at a given time.

A3. If a particular customer is unaffected by price variation, then his purchasing behavior can be explained by the simple linear learning model (SLL).

A4. Changes in price are perceived by a customer immediately, and price levels only affect customers during the particular period when they are in effect.

A5. Customers act rationally, and they all perceive Premium as "superior" to Standard. (The implication here is that if a customer exhibits no learning, i.e., if he is affected only by price, then he will almost always choose Premium over Standard when they are priced at the same level.)

A6. The effect of price variation can be explained totally by Premium-Standard price difference; actual Premium (or Standard) price level has no effect. This is a debatable assumption, but certainly evidence exists to at least support the hypothesis that price difference is the most important indicator of price effect (See Kamen and Toman [9]). This assumption is not critical and can be generalized to include a "fair price" concept (see Lilien [12]).

A7. There exists across families some (prior) joint distribution of initial probability of Premium purchase, P_0, and of price receptivity, C (roughly, the fraction of a purchaser's decision determined by price), which we will call $F_{C,P_0}(c, p)$. Families will be assumed to sample a particular c^* and p_0^* from this distribution.

A8. The parameters of the model, once chosen for a particular family, are constant over time. (Model is parameter-stationary.)

A7 details the method that has been selected for handling population heterogeneity. Thus, although it may be impossible to observe certain parameters for particular individuals, it is assumed that these parameters have some measurable distribution across the population as a whole, which one can infer as part of the parameter estimation procedure. This method of handling heterogeneity offers the sizeable advantage of not requiring the measurement of specific causal factors.

A8 implies that each family samples from some (the same) $F_{C,P_0}(c, p)$ distribution to get a particular c^*, p_0^*. These are fixed for the family. The other parameters of the model (the *SLL* parameters) are fixed across families and over time.

With the modification of *SLL* to consider pricing we introduce a controllable variable into the model and thus transform what was previously a descriptive model into a normative one. Normative models are used to determine "best" plans for realizing certain objectives. We will thus be able to explore competitive situations under different market assumptions and, with the use of *MLL* we will have the ability to determine optimal (in some model-sense to be defined) pricing behavior.

III. Model Description and Characteristics

Following A1–A8 we will be dealing here with the observation and description of the purchase pattern of a single individual (customer, consumer, family—all will be used interchangeably here) over time. Let us define a stochastic process, $\{X_t\}$, $t = 0, 1, \cdots$, to describe this behavior. $X_t = 0$ or 1 at any discrete purchase time, t, and $X_t = 1$ implies a purchase of Premium, $X_t = 0$ implies a purchase of Standard.

Let us further define the following:

δ_t = Premium price minus Standard price at t.

$P_t = \Pr\{X_t = 1\}$.

C = price consciousness of customer—roughly the fraction of his behavior determined by price.

$\phi(\delta)$ = value of the price response function when price difference $= \delta$. It will be near 1 for δ small (or negative) and will be near 0 for large δ.

Following the above definitions and A1–A8, our model (MLL) has the following form:

$$P_{t+1} = (1 - C)(\alpha + \beta X_t + \lambda P_t) + C\phi(\delta_{t+1}). \tag{2}$$

Note that δ_{t+1}, following A6, is assumed to be the sole determinant of a customer's price-buying behavior. And when $C = 0$ above, following A3, the model reduces to SLL.

When $C = 1$ the purchase is affected strictly by price and, following A5, $P_t \simeq 1$ if $\delta_t \simeq 0$ and $C = 1$. $1 \geqq C \geqq 0$, in general. We will also assume for simplicity that ϕ is continuous and has derivatives of all orders.

Note that α, β and λ defined above are constant across families. This assumption could be relaxed. λ could also be different depending on whether $X_t = 0$ or 1. In that case all the results here would have to be modified slightly. We have chosen to retain the same λ for considerations of symmetry. Sufficient conditions for this process to have a logical interpretation are that

$$\alpha, \beta, \lambda, C \in [0, 1] \quad \text{and} \quad (1 - C)(\alpha + \beta + \lambda) + C \leqq 1.$$

There exist limits here, $U(\delta)$ and $L(\delta)$, such that $P_t \in [L(\delta), U(\delta)]$ implies that $P_{t+1} \in [L(\delta), U(\delta)]$. P_t will enter this region in a finite number of steps with probability 1 if δ is fixed unless $C = 1$. When $C = 1$, real learning in the sense of this model does not occur anyway and this case will be excluded from consideration. The length of $[L(\delta), U(\delta)]$ is an increasing function of C and has length 1 when $C = 1$.

For a given value of C, there exist limits independent of δ in which P_t will be contained after finite time with probability one. Call these limits $L(\infty)$ and $U(-\infty)$ respectively.

It is easily seen that

$$\begin{aligned} L(\infty) &= [(1 - C)\alpha]/[1 - (1 - C)\lambda], \\ U(-\infty) &= [(1 - C)(\alpha + \beta) + C]/[1 - (1 - C)\lambda]. \end{aligned} \tag{3}$$

The above discussions have treated P_t as if it were constant. However, if P_t is not conditioned on past events, then it can be defined as a random variable, since it is defined as dependent on an unobserved stochastic process. If P_0 and C are known for a particular family and the price history $\{\delta_t\}$ has also been recorded, then successive purchases result merely in successive applications of (2). Then if we know T (the length of time the process has been in operation) and the parameters of the model, the value of P_t, $t \leqq T$, can be calculated directly as described in (2). We might however want to estimate P_t from knowledge of the parameters only, i.e., unconditioned on actual history. To this end, EP_t will develop. First we calculate $E[P_{t+1} \mid P_t = p_t]$ which we will denote as $E[P_{t+1} \mid p_t]$.

Recall that $P_{t+1} = \Pr(X_{t+1} = 1)$. Then it can be shown that

$$E[P_{t+1} \mid p_t, \delta_{t+1}] = (1 - C)(\alpha + (\beta + \lambda)p_t) + C\phi(\delta_{t+1}). \tag{4}$$

Repeatedly using the law of conditional probabilities, it can be shown that

$$\begin{aligned} E(P_{t+1} \mid p_0, \Delta_{t+1}) = {} & \textstyle\sum_{i=0}^{t} [(1 - C)(\beta + \lambda)]^i[\alpha(1 - C) + C\phi(\delta_{i+1})] \\ & + [(1 - C)(\beta + \lambda)]^{t+1}p_0, \end{aligned} \tag{5}$$

where Δ_{t+1} = vector of price differences $(\delta_1, \delta_2, \cdots, \delta_{t+1})$.

An interesting case to explore is that which is achieved during a period of relative price stability; that is, we can ask: what would happen if prices were to remain stable at difference level δ i.e., $\delta_i = \delta$, $i = 1, 2, \cdots, t + 1$? In this case, we get

$$\begin{aligned} E(P_{t+1} \mid p_0, \delta) = {} & [(1 - C)(\beta + \lambda)]^{t+1}p_0 + [\alpha(1 - C) \\ & + C\phi(\delta)]\textstyle\sum_{i=0}^{t} [(1 - C)(\beta + \lambda)]^i. \end{aligned} \tag{6}$$

Because of the constraints which have been imposed on the parameters, the limit in (6) as $t \rightarrow \infty$ has meaning:

$$\begin{aligned} \lim_{t\to\infty} E(P_{t+1} \mid p_0, \delta) & = [(1 - C)\alpha + C\phi(\delta)]/[1 - (1 - C)(\beta + \lambda)] \\ & = Q_\infty(\delta, C). \end{aligned} \tag{7}$$

(7) now represents the expected long-term market share for Premium purchased by an individual with price receptivity C, independent of p_0.

Now that $E[P_t \mid p_0, \Delta_t]$ has been calculated, it may be of interest to gather information about the distribution of P_t, i.e., $F_{P_t}(x)$ (independent of the observed purchase history): $F_{P_t}(x) = \Pr\{P_t \leqq x \mid p_0, \Delta_t\}$.

This distribution can be calculated explicitly for small t by summing the probabilities of the disjoint sequences which yield $P_t \leqq x$. For large t, the calculation is prohibitively complex and simpler methods are needed. See Lilien [12] for the development of such methods, and for discussion of other model characteristics—in particular, questions of model symmetry, generalization of price sensitivity behavior and combining of classes problems.

IV. Parameter Estimation Methods

Our plan of attack here is as follows: we will observe the actual frequencies that the various response strings (0110 for example) occur in the sample. We will then estimate, under the assumptions of the model, the theoretical frequency of those strings as functions of the parameters to be estimated. Values of those parameters will then be calculated to satisfy some criterion—say, to minimize a Chi Square statistic or to maximize a likelihood function. We illustrate the approach to calculating theoretical frequencies by considering purchase sequence probabilities conditional on C, P_0 for a two-period sequence with $\phi(\delta) = \phi$. Define $\Upsilon_{i,j}(c, p_0)$ = probability of sequence i, j when P_0, C are known, $i, j = 0$ or 1. Then

$$\begin{aligned} \Upsilon_{1,1}(c, p_0) & = p_0[\Pr\{X_1 = 1 \mid p_0, c, X_0 = 1\}] \\ & = p_0[(1 - c)(\alpha + \beta + \lambda p_0) + c\phi)] \\ & = (\alpha + \beta)p_0 + (\phi - \alpha - \beta)cp_0 + \lambda p_0^2 - \lambda c p_0^2. \end{aligned} \tag{9}$$

Assuming a family is randomly sampled from the population, the unconditional probability of observing sequence i, j is as follows

$$\pi_{i,j} = \Pr\{\text{sequence } i, j) = \int_0^1 \int_0^1 \Upsilon_{i,j}(c, p)\, dF_{C,P_0}(c, p). \tag{10}$$

As an example, let us observe what happens to the sequence $i = j = 1$:

$$\pi_{1,1} = (\alpha + \beta)EP_0 + (\phi - \alpha - \beta)E(CP_0) + \lambda EP_0^2 - \lambda E(CP_0^2). \tag{11}$$

Note that purchase sequences of length two cannot supply sufficient information to estimate these eight (α, β, λ, ϕ, EP_0, $E(CP_0)$, EP_0^2, $E(CP_0^2)$) parameters: two purchases yield four response categories but, since the four categories must sum to one, there are only three degrees of freedom from which to estimate parameters.

Suppose the following occurs: There are K different values of δ that were prevalent in the market over a period of time: $\delta_1, \cdots \delta_K$. Each of these time periods must be sufficiently long so that most customers will make (say) three purchases during that time.

We would then like to estimate $\phi(\delta_1)$, $\phi(\delta_2)$, $\cdots$, $\phi(\delta_K)$. Associated with this, suppose we observe empirical response proportions for three purchase strings and, in addition, calculate $\pi_{i_2,i_1,i_0}(\delta_k)$, $k = 1, \cdots, K$. Assume that each family conceptually samples P_0 and C independently from some prior distribution which we will assume here is from the Beta family; that is

$$f_C(c) = B(a_c, b_c) = \frac{\Gamma(a_c + b_c)}{\Gamma(a_c)\Gamma(b_c)}\, c^{a_c - 1}(1 - c)^{b_c - 1} \qquad c \epsilon [0, 1].$$

Then, α, β, λ, and the prior distribution of C (2 parameters) remain the same in each of the K periods. However, variation in the period-to-period prior distributions of P_0 (2 parameters per period or $2K$ parameters) must be considered to eliminate inter-period correlations and to account for marketing disturbances between periods. We then must estimate $3K + 5$ parameters: α, β, λ, a_c, b_c (for prior $F_C(c)$) plus $(\phi(\delta_k), a_p(k), b_p(k), k = 1, \cdots, K)$.

If we consider purchase strings of length 3 there are eight distinct response sequences (000, 001, etc.) yielding seven degrees of freedom per period or $7K$ degrees of freedom for estimation. If, as usual (see Cramér [4]) we lose 1 degree of freedom for each parameter estimated, K different response periods will yield $4K$-5 degrees of freedom for estimation (15 degrees of freedom if $K = 5$, e.g.). We use this information as follows:

Assuming response strings of length 3 define:

$z_{i,j}$ = Observed proportion of individuals in the population exhibiting response sequence i ($i = 1$ implies 000, $i = 2$ implies 001, etc.) during period j, $i = 1, \cdots, 8$, $j = 1, \cdots, K$.

$\pi_i(\delta_j)$ = Expected proportion of population exhibiting response sequence i during period j. This is a function of the parameters to be estimated.

N = individuals in the population.

A measure of discrepancy between these observed and expected frequencies is given by the following statistic:

$$\mathrm{X}^2 = N \sum_{i=1}^{8} \sum_{j=i}^{K} \frac{[z_{i,j} - \pi_i(\delta_j)]^2}{\pi_i(\delta_j)}. \tag{12}$$

This function is a weighted sum of squared errors with the weights being reciprocals of the expected frequencies. The expression in (12) is then minimized with respect to the parameter vector to be estimated and that value of the parameter vector which minimizes (12) is referred to as the "Minimum Chi-Square estimate" of the parameter vector.

A problem which arises here is the question of the independence of the $z_{i,j}$'s from period to period. Two cases are possible:

a. The model is true. In this case the effect of past purchase feedback is taken care of by assuming that a (potentially) different $F_{P_0}(p)$ is used in each time period for the population to sample from. Past purchase feedback dependence is thus eliminated by calculating a new set of P_0's for each period. The recalculation of the parameters for this distribution in each period eliminates the only correlation between periods, making the observations independent.

b. The model is not true. In this case the $z_{i,j}$'s may or may not be independent, depending on the real life situation. If the periods are separated by many purchase events and a long period of time, however, the natural effect of human forgetting will lead to at least approximate independence.

Assuming, then, that the $z_{i,j}$'s are at least approximately independent as above, we state the following:

As $N \to \infty$ so it can be shown that (12) is asymptotically distributed as Chi-Square with $7K - m$ degrees of freedom where m is the number of parameters (Cramér [4]). This information about the asymptotic distribution of this statistic can be used to test the "goodness of fit" of the models. It should also be noted that minimum Chi-Square estimates are consistent and efficient in this case (see Rao [26] for details).

Actually minimizing (12) with respect to the parameter vector is not possible analytically; numerical methods must be relied upon. And as (12) is not likely to be concave in the parameters, one has to search the parameter space quite carefully to be certain of a "global" optimum.

See Lilien [12] for a discussion of parameter estimation methods under less restrictive assumptions.

V. Pricing Decision Making

In this section we indicate how this model-structure might be used to aid in pricing decision making.

We are concerned with behavior in a two-brand market, where we wish to set the price of one of the brands. The competitor can be "cooperative" (wishing to maximize joint profit), "competitive" (wishing to maximize his profit), "indifferent" (as in the classical "game against nature"). He can be competitive with a different objective than ours. Or, at least, he may have a different rate of return. Our objective function may vary widely, as may our planning horizon. We may wish to look at expected average profit per period, discounted total profit, expected total profit (in a finite time) among the most common criteria. On the other hand, certain levels of market share may be important—one might wish to maximize market share or sales. Or, again, one may be concerned with a combination of, or tradeoffs in the above, or maximizing one while satisficing on one or more others. (See Ackoff [2] for discussion of objectives.) Also, the market we are dealing with may be either inelastic or price elastic in one of a number of ways.

The purpose of this listing of possibilities is severalfold: first, it creates sort of a menu from which to choose problems; second, it indicates that even in a very "idealized" situation it is not trivial to choose *the* right problem for analysis; and third, it indicates the necessity for careful selection of problems for analysis, since it is hardly possible to do them all.

We now refer back to (7) as an example of normative developments. (7) implies that there exists a limiting expected value of the probability that a family will purchase Premium which depends only on the steady state price difference and is independent of that family's initial-purchase probability. From a normative standpoint, Premium would like to know, on a total market basis, the effect of various price differences on his market share and volume.

Assume for simplicity:

1. Following the discussion of heterogeneity in §II, there exists a distribution $F_C(c)$ of price consciousness which a particular family seems to choose from. In other words, we assume that price consciousness is distributed across families in a measurable way.

2. Volume purchased by a family is independent of that family's price constant. This will permit us to identify $E_C Q_\infty(\delta, C)$ as Premium's market share. This assumption is not essential but will simplify calculations somewhat.

Under these assumptions, Premium market share can be calculated as

$$(13)\quad \begin{aligned} Q^*(\delta) &= E_C Q_\infty(\delta, C) = \int_0^1 Q_\infty(\delta, c)\, dF_C(c) \\ &= \int_0^1 ([(1-c)\alpha + c\phi(\delta)]/[1-(1-c)(\beta+\lambda)])\, dF_C(c). \end{aligned}$$

Taking two special cases of $dF_C(c)$:

1. Suppose $C = 0$, i.e., families in the market are not price conscious. Then

$$Q^*(\delta) = \alpha/(1-\beta-\lambda).$$

Here we have the result that in a non-price-conscious market, Premium's long-term market share is independent of price.

2. Suppose C is uniformly distributed in [0, 1]. Then

$$\begin{aligned} Q^*(\delta) &= \int \frac{(1-c)\alpha + c\phi(\delta)}{1-(1-c)(\beta+\lambda)}\, dc, \qquad c\epsilon[0, 1], \\ &= \int_0^1 \frac{\alpha}{(1-\beta-\lambda)+(\beta+\lambda)c}\, dc + \int_0^1 \frac{c(\phi(\delta)-\alpha)}{(1-\beta-\lambda)+c(\beta+\lambda)}\, dc \\ &= (\phi(\delta) - \alpha[1+(\beta+\lambda)\log(1-\beta-\lambda)])/(\beta+\lambda)^2. \end{aligned}$$

More complex cases of $F_C(c)$ (such as the Beta distribution) are more difficult to evaluate and may have to be carried out numerically.

An interesting problem can be formulated now: to simplify matters for the present, we will consider Standard as being non-price-competitive, i.e., Standard will set its price and then allow Premium complete freedom in setting δ, the price difference.

Define

q = unit price (assume constant over time) of Standard product.
d = cost per unit of product for Premium brand.
$V(q, \delta)$ = total market volume of the product given q and δ.

Premium's problem is to decide upon a price to set. Suppose the objective of Premium is to maximize long-term expected profit and the only strategy permissible is a single product price. Then Premium's problem reduces to:

$$\text{(14)} \qquad \text{find } \delta \text{ to} \quad \max\ (q + \delta - d)V(q, \delta) \int_0^1 \frac{(1 - c)\alpha + c\phi(\delta)}{1 - (1 - c)(\beta + \lambda)}\, dF_C(c).$$

Solution strategies for this and other, more complex problems are considered by Lilien [12].

VI. Conclusions and Uses

Our main purposes have been to (a) develop a normative model of individual consumer behavior following a certain set of postulates or assumptions and (b) explore that model's properties. A future publication will describe the performance of the model on live data.

However, the question now remains: How can a marketing manager in a particular (perhaps multi-brand) market use such a model to help set prices?

Marketing management should gather data and estimate the parameters of the model. Then, under a set of different assumptions about competitive reactions, the short- and long-term consequences of a variety of policies on the mix of individuals in the market should be explored. Only the astute executive can weigh the intangibles involved along with the quantitative aspects in order to reach an effective decision.

This may sound like begging the question; however, marketing problems are sufficiently complex (and often politically touchy) so that one cannot hope, at this stage of model building development, to do more than provide critical input for a decision. To attempt much more is to risk credibility.

How, then, should the executive put this information that he has collected to use, especially if there are many brands in the market? In many product markets, really only one brand can (or needs to) make use of this information—the price leader. Clearly a brand whose policy is to follow blindly on the heels of the market price leader can have no use for such model-building. The price leader should have a good quantitative picture of the implications of his decisions on his (and other brands') volume and profit.

Ideally, then, after estimating parameters and suggesting a policy, the next logical step is controlled market experimentation: These models might suggest policies that are outside the range of our observations. This type of extrapolation is always dangerous. To be certain that the policies we come up with are realistic, controlled marketing experimentation should be done wherever possible.

Relatively few companies are willing, however, to experiment with price. The executive may then have to be satisfied with accepting (or rejecting) the results of the models as they are and of, perhaps, running extra sensitivity studies on the relevant variables. He may thus gain added assurance that he is at least *aware* of the nature and of the magnitude of the assumptions he is making.

In summary, then, the price leader in the market should

(1) gather purchase and price data and estimate model parameters,

(2) test the fit of the model to see if it is a sufficiently accurate representation of the market situation,

(3) design an experiment to verify parameter estimates and also to infer mode of competitive reactions to price changes,

(4) run sensitivity studies on critical variables,

and then use these data to develop the type of information that can be of aid in making pricing decisions.

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